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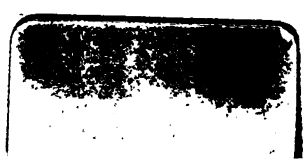

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TEACHERS' MANUAL

FOR TEACHERS USING

ARITHMETIC BY GRADES

BY

JOHN T. PRINCE, PH.D.

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PREFACE.

CONSIDERING the amount of time given to the subject of Arithmetic in our schools, it must be admitted that the results are lamentably poor. It is fair to presume that the faulty character of the text-books in general use is accountable in part for these results. With few exceptions American text-books in Arithmetic seem to leave the teacher out of the account. Definitions, rules, explanations, illustrative problems — in short, everything which should be taught by the teacher is placed before the pupil to be learned. It is true that the best teachers generally ignore these false aids and ask the pupils not to refer to them. Yet, because they are constantly before the pupils, the forbidden aids are oftentimes unwisely used, and problems are performed slyly or thoughtlessly according to the rule or model solution. While these practices are carried on to some extent in good schools, despite the care of teachers, they are universally pursued in schools whose teachers are guided by the text-book. Another fault of text-books in common use is the insufficiency of problems for practice, and teachers are compelled to make up the deficiency by placing upon the blackboard original problems or problems taken from Arithmetics other than the regular text-book. The objections to this practice are: (1) The danger for want of time of giving to the pupils unsuitable or poorly-arranged problems; (2) the harm done to pupils' eyes by close and long-continued looking, frequently before a lighted window; (3) a loss of the teacher's time in copying.

The series of books of which this Manual is a part is designed to meet the above objections. It will be readily seen that the pupils' books are not merely books of problems intended to supplement the use of an arithmetic already in the hands of pupils, but

are intended to be used independently as the only text-books needed. Moreover, the very large number and variety of problems warrant the assertion that the books contain all the problems that will be needed for drill work.

The books for pupils are eight in number, arranged somewhat on the lines of classification in City graded schools. The first two or three books are intended for use in Primary schools, the last book in advanced Grammar schools or High schools. The subjects are divided as follows:

Book I.: Numbers from 1 to 20.

Book II.: Numbers from 1 to 100.

Book III.: Integers to 1000000, Fractional Parts of Numbers, U. S. Money, Common Weights and Measures.

Book IV.: Whole Numbers unlimited, Common Fractions to Twelfths, Decimal Fractions to Thousandths, Measurements, Business Transactions, Denominate Numbers.

Book V.: Common and Decimal Fractions, Mensuration, Denominate Numbers, Business Transactions.

Book VI.: Mensuration, Denominate Numbers, Metric System, Percentage and Simple Applications, Business Transactions and Accounts.

Book VII.: Profit and Loss, Commission, Insurance, Taxes, Duties, Interest, Banking, Stocks and Bonds, Exchange, Business Accounts, Geometrical Exercises and Measurements, Ratio and Proportion.

Book VIII.: Miscellaneous questions involving the making of definitions, rules and formulas; Algebraic Exercises, Involution and Evolution, Exercises in Geometry and Mensuration, Book-keeping.

It has been the aim of the author to include in the books all subjects that are likely to be needed in any school, rather than

limit them to the possible needs of a certain class of schools. A selection of subjects, therefore, as well as a selection of problems under a given subject may be made to suit existing conditions.

The following advantages are claimed for the use of the Manual and text-books :

1. The separation of teachers' and pupils' books, whereby pupils may be taught properly and may not be given too great assistance. Suggestions as to methods of teaching and drilling, as well as the illustrative processes, explanations, rules, and definitions which belong to the teacher to develop analytically are put into the Teachers' Manual, while in the pupils' books are presented only such exercises as are needed for practice.

2. The careful gradation of problems, by which pupils acquire inductively a knowledge of arithmetical relations and principles, and skill in arithmetical processes. This is in recognition of the well-known pedagogical principles of proceeding from the known to the unknown, and from the simple to the complex. It is advised that this plan be kept constantly in mind by the teacher, and that whenever a process is not understood or is not readily performed, the pupils be taken back to processes which are well known and which can be performed readily, and then be led forward by easy steps until the desired end is reached.

3. Frequent reviews, and such an arrangement of exercises as will enable pupils to have needed practice in the applications of each principle, first by itself, and afterwards in connection with other principles which have been learned.

4. The large amount of oral work, or work which may be done without the aid of figures. Three objects of Mental Arithmetic are sought in these exercises : (*a*) Illustration of principles and a preparation for written work, (*b*) Development of the logical powers, (*c*) Cultivation of ability to work with large numbers by short processes.

5. The great number and variety of problems. The aim has been to give the *largest number* of problems that will be needed

for teaching and for drilling in all grades. For this reason, and because the forms of expression are varied, being taken from many sources, there will be no necessity of giving supplementary drill lessons on the blackboard.

6. Practicalness of work in respect to the character of the problems, and the solution of them. Care has been taken to give problems which are most likely to be met in every-day life, and to give them in a practical form. Many of the miscellaneous review problems were made by mechanics, clerks, accountants, etc., with a view of presenting conditions most likely to occur.

7. The introduction of statistics and facts of physics, astronomy, history, geography, etc., thus enabling pupils to gain incidentally much useful information.

8. The use of drill tables and other devices to save the time of teachers.

In addition to the above features, some of which are distinctively new so far as American text-books are concerned, there is the separation of pupils' exercises for practice into small books somewhat on the lines of gradation in City graded schools. By this arrangement there are gained greater convenience of handling and economy of wear than in the use of a large book which is intended to be used for several years by the same pupil.

The author is aware that the use of these books with accompanying manual in the manner proposed is not an experiment. Essentially the same plan has been pursued in Germany for many years, and it is confidently believed that American teachers will readily recognize its merits.

The work of bringing together so large a number of exercises has been materially lessened by the valuable contributions of teachers, business men, and mechanics, who generously responded to the author's request for assistance. Especial acknowledgments for such service should be made to Misses Smith, Dale, and Barber of the Practice Department connected with the Framingham, Mass., Normal School; Miss A. Roof and Mr. B. W. Drake, of Waltham, Mass.;

and Mr. L. F. Warren, of West Newton, Mass. Obligations for valuable assistance in correcting proof sheets are also due to Miss F. A. Comstock, of the Bridgewater Normal School, and Miss Amelia Davis, of the Framingham Normal School.

Suggestions for improving the work and corrections of errors, either in the Manual or Pupils' Books, will be gratefully received by the author.

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SECTION I.

GENERAL SUGGESTIONS.

Ends.— Arithmetic like most subjects of instruction may be said to have two values — its practical value as an aid in carrying on the ordinary affairs of life, and the value it has as a means of mental discipline. The ends of teaching Arithmetic therefore are knowledge and power. It is well for the teacher to know at the outset the relative importance of these ends for the purpose of making a wise selection of subjects and of knowing how they should be taught. If in our teaching of Arithmetic we keep only in mind the direct use that it has as a preparation for business, we shall find that a small fraction of the time usually given to this branch of instruction is sufficient. If, on the other hand, we consider the possibilities of a proper presentation of the various topics of Arithmetic in the cultivation of the powers of observation, imagination, reflection, and reasoning, and note the rare opportunity which the recitation in this branch has for teaching accuracy and clearness of expression, we shall be inclined to place the disciplinary side of the study far above that of the direct assistance it has in earning a living or in performing the common duties of life, and give to Arithmetic a prominent place in the Course.

Another end of arithmetical instruction which perhaps is involved in the ends already named, is skill or facility in the manipulation of numbers. The practical as well as the disciplinary value of the study is greatly enhanced by such facility as will enable the pupils to perform accurately and quickly the computations involved in the problems. It is the purpose, therefore, of good teachers of Arithmetic to lead the pupils to become accurate and quick computers, and to secure this end there is needed much practice in the application of principles already known. The

pupils can add columns, ascertain measurements, and cast interest accurately and quickly only by the repeated performance of the same mental act, and for all such work abundant practice is necessary.

Course of Study. — Although the comparative unimportance of the practical side of Arithmetic may be recognized, the mistake should not be made of spending much of the time given to the study upon unpractical subjects. The constant aim of the teacher should be to emphasize essentials, and by essentials is meant those subjects and processes which are in some degree a preparation for life or for further study. There is abundant opportunity for useful disciplinary practice in some of the common and simpler subjects, such as denominate numbers, fractions, and the applications of percentage that are made in business.

To rightly judge what the essentials of Arithmetic are, a consideration of interests other than those of direct utility is necessary. The age, capacity, and, so far as circumstances permit, the probable future occupation of the pupils should be considered. Two kinds of practice with numbers are required of pupils from the beginning of the course: one kind which has to do with the mechanical side of problems — the computations with what are sometimes called abstract numbers; and another kind which has to do with the applications and use of principles, and which involve some logical processes. The amount of the latter work is relatively small in the lower grades, and gradually increases in amount and complexity as the pupil grows in maturity, while the amount of simple computations for exactness in the four fundamental rules gradually decreases until, in the latter part of the course, nothing but concrete or applied work is done.

So far as this series of books is concerned, it is not supposed that all schools will follow the exact order prescribed, or that the books will exactly correspond with grades of pupils in all places. In the amount of work to be given also there will be a great variety of practice depending upon the age of admission of pupils, the requirements in other departments of study, and the intentions

of pupils as to the probable extent of their attendance at school. While in no one school or system of schools will it be needful to take up every subject presented, or to give to pupils all the exercises of certain subjects, it is hoped that both subjects and exercises are sufficient in number and variety to meet the needs of all, and that from them a wise selection may be made.

The Recitation. — One prime purpose of the recitation is to develop the power of self-direction in the pupils, — to lead them to depend upon themselves not only in accuracy of computations but also in all reasoning processes. For the purpose of encouraging pupils to proper effort in study, and of seeing that certain principles or processes are understood, the teacher finds it necessary to examine their completed tasks, or to test them upon what they have done. But to accomplish the objects named, it is not necessary or well to spend the entire recitation time in examination. A glance at the work performed, and a comparison of the pupils' answers with the correct answers, are generally sufficient to insure faithful effort in study. To give the lessons learned such prominence as to make them alone determine the pupils' promotion or standing in the class, is to put temptation in their way to copy solutions or answers, or to seek assistance that will not be helpful. Moreover, the detailed explanations and solutions of problems already performed, frequently mean a tedious recitation and much needless waiting on the part of some members of the class. If it is desired to ascertain whether the pupils understand thoroughly a principle or process which has been taught, a few problems which involve the principle or process may be given the class as a test. These problems should have small numbers, so that the difficulties of computation will not distract attention, and that the pupils' understanding of the question may appear in the answers they give. For example, if the subject of Profit and Loss has been taught, and the pupils have brought to the recitation the problems which they have solved in two of the cases of that subject, the teacher, after glancing at the work and leading them to correct errors in their answers, might give as a test the following problems :

A horse which cost $250 is sold for what to gain 20%?

I buy a house for $1000 and sell it immediately for $750. What was the gain or loss per cent? Suppose I had sold it for $1200. What would have been the gain or loss per cent?

From answers to these and similar questions it will be known whether the subject is understood, and time will be left for other purposes of the recitation.

When any subject or process is not understood, it should be taught, and by teaching is meant, leading the pupils to acquire knowledge by their own efforts. Little should be told the pupils in this process. The teacher's part is first to lead the pupils to recall ideas already in the mind which have some relation to the new principle or process to be taught. Then follows the presentation of the object of thought in such a way as to awaken new ideas in the learner's mind, or to readjust and reinforce old ones. The object of thought may be a physical object, as a meter-stick or a bond, or what represents an object, as a picture or diagram. Sometimes, as in teaching a rule or definition, the teacher has only to help the pupils to recall and rearrange their ideas in an orderly way, and to make a general statement from them. Some processes also may be taught by leading the pupils to recall what they have previously learned, and make the needful application. Such purposes may be accomplished frequently by skilful questioning.

When the matter in hand is taught, and ideas concerning it are clear in the pupils' minds, they need to be fixed by much repetition. This is done in the recitation by drill and in the practice which the pupils get in study. Little time is needed for teaching compared to the time that is needed for practice. A portion of nearly every recitation may well be given to drill, both upon what has been taught recently and upon what has been taught days or weeks before.

Thus it is that three means of conducting the recitation in Arithmetic should be used by the teacher: first, examination to

test the pupils' knowledge of the topic or topics; secondly, teaching to awaken new ideas in the pupils' minds; and thirdly, drilling to fix and strengthen the pupils' knowledge of what has been taught. These processes are not always separated in the recitation. Sometimes in the teaching exercise there is more or less of examination and drill, and sometimes in the examination there is necessarily much useful drill. But they all have their place in the recitation, and no one of them should be neglected.

Methods of Drill. — The effectiveness of the drill exercise will depend largely upon the methods employed. The prime object of the drill is, as has been said, to fix in the mind what has been taught. To gain this object, there should be a repetition of the mental act which was occasioned when the principle or process was taught. In the drill exercise, the teacher should have in mind exactly what acts of the mind are to be repeated, and he should also aim to have every pupil employed during the exercise. Drill which has not a definite object, or which consists of a repetition of words only, or which permits inactivity on the part of some members of the class, cannot accomplish all the desired ends.

The drill may be both oral and written. If oral, the given problem may be read by one pupil, all the rest of the class reading it silently at the same time; or it may be dictated by the teacher. In either case all should solve the problem together, either following the oral solution of a pupil, or else performing it silently and indicating the answer on slate or paper. By the latter plan it would be well to have all the pupils work and write very promptly exactly at the same time. For example, the pupils have before them paper or slate and pencil. The teacher says, "pencil in hand," and dictates (not from a book) the problem. After giving sufficient time for all to solve it, the teacher says, "write the answer," and then after just enough time for the pupils to write the figures of the answer, says, "pencils down." The teacher then gives the answer, and asks all pupils who have that answer to hold up the slate or paper so that the teacher can see the answer. It will be necessary for the figures of the answer to be written large

and in a given place, that they may be seen readily. Properly managed, this method of drill will afford opportunity for giving much practice in a short period of time. This method will serve for examination as well as for dictation.

Written drill work may be given from dictation or from the book. If from the book, a number of exercises may be given out for pupils to perform as many as they can within a certain time, or the exercises may be given singly to members of the class, each pupil performing such problems as are most needful or most useful for him to perform.

Reviews. — For the sake of making pupils familiar with certain important principles or processes, and also for the sake of connecting together kindred subjects, frequent reviews should be made. In giving such reviews, let the teacher have in mind a well-defined purpose, and not force pupils to work simply for the sake of working. No exercise should be given that is not needed, either for fixing in the mind what has been taught, or for testing the pupils' knowledge of a given subject, or for securing skill and facility in the use of numbers.

Tasks and Desk-work. — The lessons set for practice or study should, as nearly as possible, serve the needs of every pupil in the class. So far as the character of the work is concerned, those subjects only should be given for tasks which are really needed to be reviewed or which will serve as a test of the pupils' knowledge. The given lesson also should be of sufficient length to employ *all* the pupils of the class a reasonable amount of time. To accommodate the work to the different degrees of ability and quickness in members of the class, it may be well to give a lesson which all can do, and in addition, extra work for those who are able to do more.

Illustrations. — Problems involving measurements, and other problems whose conditions are not quite clear, should be illustrated by means of plans or diagrams. Care should be taken, however, that the attention of pupils be not diverted from the real purpose of the drawings by their nicety or elaborateness, and that illustra-

tions be not required when the problems are fully understood, or when the problems can be performed without them.

Reading Problems. — Let the pupils form the habit from the outset of reading over thoughtfully every problem before they begin the solution. The points to be noted in the reading are: What is required? What steps are to be taken? About how large will the answer be?

Analyses and Explanations. — Oral analyses or explanations in the lower grades should be very simple. In these grades short and direct statements as to what has been or is to be done in the solution of a given problem may be made. The statements should as nearly as possible represent exactly the pupil's thought and should consist therefore of words largely of his own choice and arrangement. In the middle and upper grades more and more minute analyses may be insisted upon, the pupils being required as their minds develop to give reasons for the steps taken. But in no part of the course should pupils be required to give formal and ready-made explanations which do not represent their own thinking. Some statements which explain to the teacher may actually bewilder the pupils and deaden their thinking powers.

Problems and Indicated Operations. — From a very early period in the course in Arithmetic the pupils should be led to see and to indicate operations that are involved in problems. In the first year pupils are encouraged to make story problems from equations and soon after to make the equations from the story problems. This work should go on all through the course until in the later grades much of the work in Arithmetic may well consist of written or oral statements of processes that are involved in given problems.

Forecasting Answers. — Every teacher knows what surprising and unreasonable answers to problems are given by pupils, being in some cases many thousand times too large or small. To prevent such mistakes and to encourage thoughtful attention to the conditions of problems, it is well by means of questions first to give an approximate estimate as to what the answer will be. Suppose for example the problem is: "How many bushels of potatoes will

be gathered from a lot of land 100 feet square if it yields at the rate of 200 bushels to the acre?" The teacher may ask: "How many sq. ft. in the lot? What part of an acre in the lot? If the lot is a little less than one quarter of an acre, less than how many bushels will the lot yield?" Or if the problem is: "How many bushels of wheat can be put into a bin 6 ft. 4 in. long, 4 ft. 2 in. wide and 3 ft. deep," the teacher may ask: "About how many cu. ft. does the bin contain? How do a bushel and a cubic foot compare? Will the bin contain a few more or less than 72 bushels then?" Gradually the teacher may give less and less assistance in this direction until the pupils will be able to forecast the answers independently and form the habit of so doing.

Language. — The good teacher is quick to see the useful and constant opportunities which Arithmetic affords for teaching language. From the very first year when children are encouraged to tell little number stories to the highest grade of the Grammar School when explanations, rules and definitions are required to be made by the pupils themselves, is there constant occasion for the clear and accurate expression of thought. The very accuracy of the thought required in Arithmetical processes and definitions makes accuracy of expression necessary both in the choice and in the arrangement of words.

SECTION II.

FIRST STEPS IN NUMBER.

Before the book is put into the hands of children, it will be well to teach the numbers to ten by the aid of objects without the use of figures. This may occupy a greater or less time according to the ability and previous training of the children. Most children who enter school at five years of age know very little of number, while others who enter at a later period, or who have had the

advantage of a good inheritance or helpful home and kindergarten training, can recognize numbers and make all combinations and separation of numbers to four or six. For the sake of a proper adaptation of the work to the capacity of the children, the separation of the children into groups will be found advisable. The division made need not be fixed. There is as much difference among children in the growth of their intelligence as there is in the intelligence itself. Accordingly pupils should be transferred from one group to another as soon as they are discovered to be ahead of or behind their fellows.

Counters. — On some accounts the variety of objects used for teaching numbers in the first lessons should not be great, being limited to the splints or wooden blocks. If blocks are used, they might be either of three sizes, viz.: cubes ($1 \times 1 \times 1$ in.), half-cubes ($1 \times 1 \times \frac{1}{2}$ in.), quarter-cubes ($1 \times \frac{1}{2} \times \frac{1}{2}$ in.). The whole cubes are preferred by many teachers. Splints have the advantage of being easily handled.

Methods. — For teaching number to young children, there should be a table about which the children can stand in full sight of the counters in front of the teacher, who sits at the head of the table. Or, if there is no table, the children may sit in their seats and the teacher use an inclined table in sight of the children.

If the number three and their combinations are known by the children, ask them to place three blocks together. After this is done by all, the teacher and children put one block with the three blocks, and the new number is named by the teacher. *Teacher:* Three blocks and one block are how many blocks? *Children* (answering singly): Three blocks and one block are four blocks. *T.* (taking away one block): Four blocks less one block are how many blocks? *Ch.* (taking away one block as the teacher did): Four blocks less one block are three blocks. *T.* (moving the blocks): Two blocks and two blocks are how many blocks? Four blocks less two blocks? One block and three blocks? Four blocks less three blocks? Two twos of blocks? One four of blocks? One-half of four blocks? One-fourth of four blocks?

How many twos of blocks in four blocks? How many ones of blocks in four blocks? Answers to these questions are to be given in order, the objects being used in the manner indicated. The work with objects should be continued until the children have a clear and definite idea of the combinations. These exercises may be followed by a comparison of four blocks with three blocks, and two blocks and one block, the children being led with the groups before them to answer the following questions: Four blocks are how many more than three blocks? Three blocks are how many less than four blocks? Four blocks are how many more than two blocks? Two blocks are how many less than four blocks? Four blocks are how many more than one block? One block is how many less than four blocks?

In the same way numbers to ten may be taught, no abstract memory work being required or expected during these first lessons. The following facts of numbers to ten should be taught:

5: { 4+1; 5−1; 3+2; 5−2; 2+3; 5−3; 1+4;
5−4; 1 five; 5 ones; 5÷1; ⅕ of 5.

6: { 5+1; 6−1; 4+2; 6−2; 3+3; 6−3; 2+4;
6−4; 1+5; 6−5; 1 six; 6 ones; 6÷1; 3 twos;
6÷2; 2 threes; 6÷3; ½ of 6; ⅓ of 6; ⅙ of 6.

7: { 6+1; 7−1; 5+2; 7−2; 4+3; 7−3; 3+4;
7−4; 2+5; 7−5; 1+6; 7−6; 1 seven; 7 ones;
7÷1; ⅐ of 7.

8: { 7+1; 8−1; 6+2; 8−2; 5+3; 8−3; 4+4;
8−4; 3+5; 8−5; 2+6; 8−6; 1+7; 8−7;
1 eight; 8 ones; 8÷1; 4 twos; 8÷2; 2 fours;
8÷4; ½ of 8; ¼ of 8; ⅛ of 8.

9 : $8+1$; $9-1$; $7+2$; $9-2$; $6+3$; $9-3$; $5+4$; $9-4$; $4+5$; $9-5$; $3+6$; $9-6$; $2+7$; $9-7$; $1+8$; $9-8$; 1 nine; 9 ones; $9 \div 1$; 3 threes; $9 \div 3$; $\frac{1}{3}$ of 9; $\frac{1}{9}$ of 9.

10 : $9+1$; $10-1$; $8+2$; $10-2$; $7+3$; $10-3$; $6+4$; $10-4$; $5+5$; $10-5$; $3+7$; $10-7$; $2+8$; $10-8$; $1+9$; $10-9$; 1 ten; 10 ones; $10 \div 1$; 5 twos; $10 \div 2$; 2 fives; $10 \div 5$; $\frac{1}{2}$ of 10; $\frac{1}{5}$ of 10; $\frac{1}{10}$ of 10.

Picturing of Problems. — The picturing of problems may sometimes be given for busy-work in the lower grades, one object being to provide a pleasant inducement for the children to review what has been taught. This suggests the advisability of teachers' showing pupils how the drawings may be made and allowing them a certain degree of freedom for the exercise of their inventive powers. But care should be taken not to carry the picturing too far. It should be remembered that such work is a means and not an end. Properly used, the representations will serve to make clear the relations which are not fully grasped and to fix in the mind combinations and processes which are not quite clear; but carried to the extent of diverting the attention from the main purpose of the exercise, they are to be deprecated.

Use of Figures. — Early in the year figures to represent numbers should be used, first in having the children read what is written by the teacher, and afterwards in writing the figures. Such work may proceed in the following order: (1) The objects counted and placed in a group by each child. (2) The name of the number given by the teacher and repeated by the children. (3) The figure written on the board and read by the children. (4) The figure copied or written from dictation on slate or paper by each child. The teacher should be careful to present only good models before the children for imitation.

The writing of equations to represent combinations of objects

and the interpretation of represented combinations constitute a useful feature of number work. For example, the children are told to put 3 sticks with 3 sticks, and after telling how many in all, the teacher writes on the board $3 + 3 = 6$, which the children read. The same is done after all operations with sticks in addition, subtraction, multiplication and division. At another time the teacher writes on the board an expression, as $4 + 2$, and asks the children to do with the sticks what that says. This exercise will lead to a good kind of busy work in number.

Number Stories.— For purposes of language as well as for the purpose of affording a pleasant means of drill in number, pupils should be led to tell stories involving the combinations which have been taught. The general plan of such work may be as follows: First let the teacher make a story, each pupil using the sticks as called for. Then each pupil may tell a story and as he talks, he, with the other members of the class, uses the objects for illustration. After this has been done with the aid of sticks, stories may be told from spoken or written examples or equations, and finally they may be told with no assistance or suggestion from any one. In all this work the aim should be to make it so interesting that all members of the class will pay close attention. The following lesson will illustrate the plan above given:

Teacher. Show me six sticks. Show me four of them. How many sticks are four sticks and two sticks? Call the sticks books. *Children.* Four books and two books are six books. *T.* I will tell a story and as I tell the story you may work with the sticks. I have four apples and my brother has two apples. How many apples have we both together? *C.* Both have four apples and two apples. Four apples and two apples are six apples. *T.* Who can make a story about five and one? Mary. *M.* I had five cents and my mother gave me one cent. How many cents have I now? *T.* Charlie may tell. *C.* You have five cents and one cent. Five cents and one cent are six cents.

(The "story" may be given in the form of a question, as above, or the children may be led to give the solution in a statement,

thus: If I have 5 cents and my mother gives me 1 cent more, I have 5 cents and 1 cent. 5 cents and 1 cent are 6 cents.)

T. Who will tell a story about 3 twos? Jamie. *J.* If I buy 3 pencils and pay 2 cents apiece for them I pay 3 times 2 cents. 3 times 2 cents are 6 cents. *T.* Willie, another. *W.* 3 rabbits have 3 times 2 eyes. 3 times 2 eyes are 6 eyes.

In all this work it is supposed that every member of the class is following with or without the objects the stories as they are told. More elaborate stories may be told and more extended combinations may be made if the attention can be held. Such questions as the following may not be too difficult provided the objects are used and progress is made by easy steps.

Charlie was given 10 cents to buy something for his mother. He bought 2 oranges at 3 cents apiece and a yeast-cake for 2 cents. How many cents did he bring back?

I had 6 cents and my brother gave me 2 more. With the money I had then I bought apples at two cents apiece. How many apples did I buy?

From given stories children may be led to perform the required operations with objects and to make the proper equations. The finding and writing of equations from problems constitute an important part of number work later in the course and may well be begun in the lowest grade of schools.

SECTION III.

NOTES FOR BOOK NUMBER ONE.

(The numbers at the head of the page indicate the number and page of the Pupils' Book for which notes are made: thus [II. 18] means Book No. 2, 18th page. Figures in heavy type on a page of the Manual denote the page or pages of the book for which notes are made.)

The exercises of Section I. are intended as a review of what has been taught with objects. It may be necessary still for objects to be used occasionally, both by pupils at their seats and by the

teacher and pupils in recitation, but the aim should be to lead pupils to perform promptly the exercises of this section without objects before the second section is begun. If an example or problem is not readily performed, the want of promptness is due to one of two causes: either the pupils do not understand the operation involved, or else, understanding it, they have not facility in performing it. If an operation is not understood, it should be taught, and the best method of teaching ideas of numbers and their operations is by the use of objects. Facility is gained through continued practice in repeating the operation in which facility is needed. This practice may be gained in the recitation and in the busy work of the pupils at their seats. If the previous lessons have been well taught, it is supposed that pupils need only the drill of busy work and recitation practice in exercises of Section I. to prepare them for the work of Section II.

For counters use the blocks or wooden sticks which have been used previously (see Manual, page 9), or small pasteboard squares which can be made with little difficulty. If the recitation table could have upon it lines indicating squares of the size of the counters, some advantages for placing and comparing the counters would be gained.

The use of lines and the rough drawings of objects in performing problems is shown on pages 20 and 26. Such work should be used constantly in the lower grades, care being taken that in representing numbers and operations by drawing the attention of children be not diverted from the real purpose of the exercise, either in the complexity of the representations or in their extreme accuracy or nicety.

Other suggestions as to the scope and use of the book will be found in the "Note to Teachers," page 1.

2–11 The design of these exercises is to present graphically before the children the various combinations and to give a pleasant means of reviewing what they have had with objects. Let the children practice upon the illustrated pages until the combinations can be given quite freely. The sign of multiplication as used in

these books is supposed to be read "multiplied by" rather than "times" for two reasons: 1st, for the sake of uniformity. The signs + — and ÷ all indicate that the number preceding the sign is to be operated upon, 2 + 3 means that 3 is to be added to 2 and that the expression should be read 2 and 3 or 2 plus 3; 6 — 4 means that 4 is to be taken out of 6 and it should be read 6 less 4; 8 ÷ 4 means that 8 is to be divided by 4 and should be so read. In the same way 4 × 2 means that 4 is to be multiplied by 2 and that it be read 4 multiplied by 2. Another and stronger reason for placing the multiplier after the sign is convenience in the analysis of problems. This will appear later when the written analyses of problems are made by the pupils. If desired the expression 4 × 2 may be read 2 times 4.

The pages opposite the illustrative work are designed for busy-work and for recitation. In recitation the children should be led to read the sentences supplying the ellipses as they read.

12–15 These exercises are to be used for busy-work and also for recitation. If necessary the children may be permitted to use the splints in finding the answers, but in the recitation they should be expected to give the answers promptly after the statement, thus "4 and 5 are 9," "3 and 5 are 8."

16 A good exercise is to put pairs of numbers on the board as here indicated and have the children give the sum instantly. This will assist in subsequent work in adding columns.

17 The children should be encouraged to make illustrations of processes as here indicated.

18–19 The children with a little assistance can be led to make original stories like the following: "My father had 8 ducks and he sold 4 of them. He had then 4 ducks left."

20 Good illustrative work is here shown which the children can occasionally do by themselves for busy-work.

21–25 A variety is here given for drill both in busy-work and recitation.

26–28 With little effort on the part of the teacher, the children can be led to illustrate problems for themselves. But in

all this work observe the caution given on pages 11 and 14 of the Manual.

29–36 Let the teaching of each number above 10 precede the recitation from the book. Thus in teaching 11, place upon the table in sight of all the pupils 10 square blocks in a column. The children who are sitting at their seats or standing around the table do the same with their squares. The teacher asks how many blocks there are and proceeds as follows: "We will place one more block with the ten. Now we have eleven blocks. How many blocks have you, Charley? Eddie? etc. How many blocks are ten blocks and one block, Mary? Susie? etc. Take away one block and how many have you? Eleven blocks less one block are how many blocks?" Other facts in 11 developed in the same way are shown on page 29. Where there are two subtractions, as in $11 - 5 - 5$, lead the children first to combine $5 + 5 + 1$, leaving the blocks as indicated, and then to remove a little way from the collection 5 blocks, covering them up as they say: "11 blocks less 5 blocks are 6 blocks"; repeating the process they say: "6 blocks less 5 blocks are 1 block, 11 blocks less 5 blocks less 5 blocks are 1 block." The groups of blocks are now in position to divide by 5, the question being: "In 11 there are how many fives and how many left over?" or, "5 blocks in 11 blocks how many times and how many left over?" After the separation of the number into fours, threes, and twos, a comparison of 11 with all numbers below should be made; thus, in answer to questions, the children are led to say: "11 blocks are 1 more than 10 blocks; 11 blocks are 2 more than 9 blocks, etc. 11 is 1 more than 10," etc.

The writing and reading of numbers should be carried on in connection with objects. Sticks may be the most convenient objects for this purpose. The teacher takes 11 sticks and after asking the children how many there are, binds 10 of them together with a rubber band and says: This is 1 ten of sticks. How many tens of sticks in eleven and how many more? The children should be led to bind the 10 sticks into a ten as the teacher has done and to say that in 11 there is 1 ten of sticks and 1 more. To express

in figures this number the ten bundle may be placed on the ledge of the blackboard and the children be led to write 1 above it for 1 ten. The 1 stick may be placed beside the ten and another figure 1 be placed above the 1 stick, making 1 ten and 1, or 11. This exercise should be worked out very slowly and carefully, and if necessary it should be repeated several times, the purpose being to lead the children to have a clear idea of the decimal system at the outset.

After this work with objects has been done, equation statements may be given by the pupils in answer to questions given by the teacher or read from the book. Thus all operations that have been performed with the objects are now given orally without objects. Then follows the writing of the equations by the children. This will constitute a part of the busy-work of the children, who copy from the book the questions and complete the equations. The story problems which naturally follow may be told and solved in the recitation and also at the seat.

The order of procedure in all this work should be noticed—(1) operations with objects, (2) giving of oral equations, (3) giving of written equations, (4) giving of story problems. Sometimes drawings may be made in illustration of operations directly after the objects have been used, and sometimes the drawings may be used instead of the objects. For review, the order above given may be changed or inverted. The oral or written equations may be given by the children, who will illustrate or demonstrate the equations by the use of objects or drawings ; or the children may give the story problem, afterwards solve it by objects or drawings and finally make the oral and written equations.

The order above given with such variation as circumstances will determine, should be followed in treating each new number.

37–39 Nearly all the questions on these pages should be solved by means of simple drawings as shown on page 37. Some care will have to be taken to show the children just what is expected of them. Slow progress in picturing problems at this stage must be expected. From what the teacher does upon the blackboard in

illustrating a process, and from little corrections that are made, the children will gradually get the idea of representing the conditions and the solution of problems by means of drawings.

40 In teaching 13, first use the squares and blocks in the order indicated, somewhat as is given on pages 29 and 33. With some suggestions and directions, the children can be led to draw the squares and to indicate what process is shown. Dwell upon this page until a tolerable degree of familiarity with the combinations is had.

41–43 In reciting these exercises the children may read only the answer thus: in *a*, page 42, "Twelve boys and one boy are thirteen boys," etc.; in *e*, "In twelve there are 6 twos," etc.; and in *g*, "I must put ten with three to make thirteen." In the story problems let the children first read the problem and then give the answer in a good statement, thus: in *f*, page 43, "The little boy's sister found eight eggs and three eggs, or eleven eggs."

44 A teaching exercise with the coins should precede the recitation of these problems. 10 one-cent pieces, 5 two-cent pieces, 2 five-cent pieces, and a dime will be needed to show all the facts given. The children already have some knowledge of the value of these coins, and the exercise with them may be made very interesting. Thus, the teacher may say, "Who will find six cents from these coins?" "Charlie has a dime. Who will take from the two-cent pieces enough to have as much money as he has?" The children will also enjoy buying, selling, and making change with the coins.

45 The combinations indicated on this page should be taught first with objects, and then shown by squares as given on pages 29 and 33. The children should also be expected to make the drawings with as little assistance as possible.

46–47 In the column addition, lead the children to add in pairs, thus: in Ex. **4**: 6, 8; 7, 11, etc.; and in Ex. **5**: 6, 13; 8, 12, etc.

49 First teach with objects as before. The answers should be given with and without the aid of squares.

51–52 The form of problems and answers in Exercises **12** and **13** is a model for original problems which should be given by the children occasionally. These may be given with and without objects.

54 The problems involving days in the week, weeks in the month, and units in the dozen, should be extended by the teacher and by the children with as great variety as possible.

55 Introduce the measures here, using water or dry sand. Let the children measure the water or sand until the facts become fully understood. The measures should also be used in teaching problems similar to those in the lower part of this page. This is followed by picturing the problems as shown in the cut. The children should make the drawings with as little assistance from the teacher as possible. Such work will be found very useful and interesting to the children. The illustrations should always be made whenever the children find difficulty in making a mental image of the conditions, and in performing the problem. Three or four days may be profitably spent upon problems similar to those given on this page and the page following.

57 The measuring strip called for should be cut from stiff paper; pasteboard or cardboard would be better. If the children cannot cut and mark this strip accurately, the teacher or older children should do it. Several exercises with this measure and a yard-stick, similar to those on the lower part of the page, should be given.

58 A teaching exercise should precede drill upon this page. Teach rectangle and square, by showing that rectangles have square corners and that squares are rectangles whose sides are equal. Paper cutting and folding should accompany the solution of *c*, *d*, *e*, *f*, and *g*. Other similar problems may be given.

59 If the children cannot perform these problems readily, objects and drawings should be used. Let the answers be in entire sentences, thus: "In two quarts there are four pints," or "There are four pints in two quarts." Do not insist upon one form of answer to these problems. If the statement of the

pupil expresses a clear thought of the number combinations let it pass.

62 The children should be encouraged to illustrate these problems in as many ways as possible. After the processes have been shown by drawings, the problems should be read and full statements of answers should be given. Here also one particular form of statement should not be insisted upon.

64 Some of these problems, such as *d*, *f*, *g*, and *h*, should be taught before they are given to the children. Add by pairs as well as singly, numbers indicated in the columns. Thus the first column may be added, 6, 8, 11, 13, or 6, 11, 13.

68–69 While a particular form of answer should not be insisted upon, the answers given should be corrected if they are not clear. A good form may be given by the teacher, as in *a*, page 68: "If I have a dime and a five-cent piece I shall have fifteen cents, and I shall need two cents more to have seventeen cents"; and in *h*: "Two boys have four legs and three dogs have twelve legs. Two boys and three dogs have four legs and twelve legs. Four legs and twelve legs are sixteen legs." If the children can write without a copy, it may be good exercise in written language for them to write the answers as they have been accustomed to recite them in the class.

70 In teaching to write each new number, pay particular attention to the expression of the ten and the units as shown on page 16 of the Manual.

72 These should be given *at sight*. In recitation do not permit the children to read the question first and then give the answer by repeating a part of the question. The answers should be given in entire statements, with no pause before the answer. Occasionally the children might give the answer quickly to each question by a single word.

73 Sight exercises for drill. Let the children give results promptly, thus: 7, 10, 8, 5, etc.

74 Let the answers to these problems be given in entire sentences. The solution of some of them may have to be pictured before they are understood by the children.

78 Show by bundles of sticks the expression of two tens and no units.

81 A portion of this page only may be given at a time. Observe the order of representation. Review frequently what has been taught.

84–86 The problems of these pages will suggest a kind of work that may be done in connection with nature lessons and language. Lead the children to give the answers in entire sentences. Thus, in **26**, page 84, the answers may be: (*a*) 3 pansies have 3 times 5 petals. 3 times 5 petals are 15 petals. (*b*) I pay 6 times 2 cents for 6 two-cent postage stamps. 6 times 2 cents are 12 cents. I pay 5 cents for the paper. I pay 12 cents and 5 cents for the stamps and paper. 12 cents and 5 cents are 17 cents.

Let the statements be made by the children in reply to questions. Gradually they will be able to give the answers in proper form without assistance.

SECTION IV.

NOTES FOR BOOK NUMBER TWO.

For a brief statement of the purposes of this book and for some general directions see the Note to Teachers given in Book No. 2. It may be said further, that objects should be used in teaching every new fact or process. The illustrative work here given is intended to supplement and not to take the place of such teaching. The objects needed will be pasteboard squares and sticks for counters, common weights and measures, and toy money. The kind of illustrative blackboard work which may be done by teachers will appear in the notes.

After the various operations have been taught, much drill involving a repetition of the mental act in learning them will be found necessary. There is given, accordingly, in this book a great variety and number of drill problems. For variety of class-work

it may be well to give exercises from the blackboard from time to time, varying with the needs and progress of the children, but for seat-work it is believed that the given exercises will furnish abundant material.

1–11 These problems are a review of work given in Book No. I. If the children cannot perform them with some degree of promptness, it is advised that needed portions of the previous book be taken up. Possibly there will be needed only some oral or blackboard drill. It may be well to place upon the board the following for drill in addition:

1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	9
2	2	2	2	2	2	2	2	
2	3	4	5	6	7	8	9	
3	3	3	3	3	3	3		
3	4	5	6	7	8	9		
4	4	4	4	4	4			
4	5	6	7	8	9			
5	5	5	5	5				
5	6	7	8	9				
6	6	6	6					
6	7	8	9					
7	7	7						
7	8	9						
8	8							
8	9							
9								
9								

These figures represent all possible pairs of units that can be made. There is no combination in simple addition that may not be referred to them. It is for this reason that a thorough knowledge of these combinations should be had.

12–13 If necessary, before giving these lessons from the book, teach the writing of numbers to 20 with the aid of sticks as suggested on page 17 of the Manual. Further drill similar to that given on page 12 may be given.

14–19 The teaching of numbers to 100 by means of sticks should precede and accompany these exercises. The purpose of such teaching is to give the children a good foundation knowledge of numbers and their expression. As before, every ten of sticks should be bound into a bundle, and the number of such bundles in a number should be called so many tens, while the single sticks should be called ones or units. For example, if twenty-six is the number to be taught, let the teacher count out ten and place a rubber band about it, proceeding as follows: "How many sticks have I here? We will call it a what? Yes, a *ten*. Let us count out ten more. How many tens have I now in my hand? How many more sticks have I? How many sticks in all? Two tens of sticks and six sticks are how many sticks? Now let us write this number on the board (placing the two bundles together and six sticks together on the ledge). Who will place above the tens bundles the figure which tells how many tens? Who will place above the single sticks the figure which tells the number of ones or units? Who will read the whole number?" The same method should be pursued with other numbers in the twenties and thirties, and then the children should be given sticks and bands to count out and express given numbers as far as forty. The children are now ready to do the work called for on page 14. The same course should be followed in teaching the numbers to 100. Several days may be profitably spent upon this work, the book being used for review in the recitation and for busy-work. Lead the children to draw the squares very carefully with a ruler or other straight edge.

20 A review which should be performed without objects.

21–26 Observe carefully the order of these problems, taking up new work only when the old is well understood. Lead the children to perform the work of addition and subtraction first with

sticks, afterwards by means of drawn squares, and finally without the aid of objects. Do not proceed too quickly to the abstract work because the children can give answers to the questions. One important purpose of using objects at this stage is to give a good foundation for subsequent work.

The problems on page 23 are intended for sight-work, but it may be well to have the children repeat the problem first before giving the answer; thus: Twenty and ten are thirty, and four are thirty-four; thirty and twenty are fifty, and three are fifty-three, etc. Fifty-six less twenty are thirty-six, etc. If the children hesitate in giving answers, go back one step; for example, if they cannot give an answer at once to the question $53 + 40$, give problems in the addition of tens alone, $30 + 20$, $40 + 30$, $50 + 40$, etc. The same course should be pursued in subtraction.

The questions on page 24 should also be repeated thus: Five and two are seven, fifteen and two are seventeen, etc. Adding so as to make even tens, and subtracting so as to break up even tens, should be fully shown by objects and drawings. Possibly similar work will have to be given by drill from the board.

27–28 The children should read these problems silently, and repeat a portion of them in the answer; thus in **4**, page 27, the answer might be: "If I sleep ten hours of the day I shall be awake twenty-four hours less ten hours. Twenty-four hours less ten hours are fourteen hours." Of course other forms of answers should be permitted.

29–31 These examples *can* be performed by the children without objects, and they will not be found very difficult; but it will be far better to use the sticks for a day or two upon similar examples, both for the expression of the numbers and for the process of adding or subtracting. This will be especially necessary in subtraction. Thus to take 60 from 84, the children take 8 tens of sticks and 4 sticks, and express that number in writing as 84. They are then asked to take 6 tens of sticks or 60 sticks from the number. They do this and find 24 sticks remaining. This process they represent by figures, writing one number under

the other, drawing a line under the lower number, and writing the remainder below. All this is a good preparation not only for the problems of these three pages but also for the problems which follow.

32–35 Here is taken a new and important step in addition and subtraction — the adding of units whose sum is more than ten, and the subtracting of units from a ten and units, the units of the whole being less than the units of the part. The children have learned to perform these operations to twenty, and now they are to apply the knowledge thus gained to larger numbers. Sticks should be used as before, first by the addition of 2 to 9, making 1 ten and 1 unit, and afterwards by the addition of the same number to 19, 29, 39, etc. Let the children stop in each case to put the band about the new bundle of ten, and express in figures what they have done. The same course should be taken in subtraction. The book may now be taken for the solution of problems by the aid of the cuts. The drawing of the squares in the manner indicated on pages 32 and 34 will be good busy-work for the children. The answers may first be given by repeating the numbers to be added or subtracted, and afterwards at sight by giving the result alone; thus, in **3**, page 33 : Eleven less two are nine, twenty-one less two are nineteen, etc. Nine, nineteen, twenty-nine, etc. The children will be able by repeated drill to add and subtract by twos and threes very rapidly.

36–46 There is no new principle involved in the problems of these pages, and the same course should be followed in teaching and drilling which was pursued in the previous four pages. Do not neglect to use the objects and drawings whenever a new number is to be added or subtracted. A repetition of the mental act which accompanies the work with objects is as necessary as the repetition of the mental act in recalling former impressions. Very much drill is needed, both with and without objects, to fix the combinations in the minds of the children, so that they will be given with perfect accuracy and promptness. Occasional drill-work on the board may be found useful. For busy-work, lead the children to

write out problems in the addition and subtraction of 5, 6, etc., in the manner indicated on page 37. This will aid them in future written work.

47 A simple extension of the process already learned is required here. Use objects and drawings of lines, dots, or squares. In the cut on this page let the children place a card over the squares in such a way as to show 2 threes, 3 threes, 4 threes, etc. Read in **6**, Two multiplied by four, or four times two.

48 The getting of fractional parts of numbers by the aid of objects and drawings is very interesting and useful work for the children, and should be frequently done. This objective work should be immediately followed by a repetition of the solution of the problem without the objects. For example, after the children have learned that $\frac{1}{3}$ of 12 dots is 4 dots, and that $\frac{2}{3}$ of 12 squares are 8 squares, they should be asked the questions, "What is one-third of twelve? two-thirds of twelve?"

The division of numbers by numbers is the reverse of multiplication, and may be taught directly in connection with it. After counting out 24 sticks, have the children separate them into groups of 3. Then ask, "How many threes of sticks are there in twenty-four sticks?" or, "Three sticks in twenty-four sticks, how many times?" The same may be done with the drawn squares, letting the column or line of squares represent the divisor, and the uncovered squares the dividend.

49–51 Partition and division by 4 are to be taught and drilled upon as was indicated for the same processes by 3. Before the solution of applied problems, the children should be able to give with a considerable degree of promptness answers to all examples on these pages.

52–54 The most difficult of these problems should be taught with objects in the recitation, but in teaching them be careful not to do anything for the children which they can do themselves. For example, in **8**, page 52, you say first, "We will let the sticks represent eggs, and you may count out one dozen. How many eggs have you, Willie? How many have you, Mary? Now, if my

hens lay four eggs every day, how many days will it take them to lay the dozen." Do not disturb the children in their thinking, until you see that the process is not clear. If it is not clear say: "If they lay four eggs in a day count out how many they would lay in two days. Now see if you can find how many days it will take the hens to lay a dozen." Probably no further help will be needed. Then proceed with the larger number required, varying the number, and calling upon children singly. After the most difficult of the problems are taught, let the children "picture" them, and others which have not been taught. Thus, in the above problem, the children could be led easily to illustrate it as follows:

and to write below the pictured solution: "It will take 3 days for the hens to lay a dozen eggs if they lay 4 eggs a day." If it is thought best to have the representation of objects more realistic, ovals instead of lines could be drawn.

After the problems have been taught and pictured, the children should be ready to solve them without aids, and give the answers in entire sentences, as in **5**, page 53: "From the fourth to the twenty-second of June there are eighteen days. In eighteen days there are two weeks and four days."

55–56 Not much time is needed for the multiplication and division by five, and yet it is not well to neglect any part of the objective work. The addition and subtraction work indicated on page 55 may be extended if it is found that the children are losing their hold of such work.

57 It will probably not be necessary to teach any of these problems. If not, give them to the children as they are, to work out with and without drawings.

58–59 In teaching and drilling, dwell particularly upon those examples which are most difficult to remember, as for example, $6 \times 7, 6 \times 9, 42 \div 6, 54 \div 6$. For review in addition and subtraction

as well as in division, practice considerably upon such examples as **10** to **13** and **16**, page 59. This may be done by giving a number, as 38, which the children are expected to divide by 6 or 5. The answer would be, "There are 6 sixes and 2 in 38," or, "6 is contained in 38, 6 times and 2 remainder."

60 If the previous work has been thoroughly done, the children will find little difficulty in performing these problems according to the model shown in **1**.

61–62 7×6, 7×8, and 7×9 are likely to give the children most trouble to remember. Drill upon these combinations, therefore, should be most persistent. The other multiplications by 4 and 6 have been learned previously, with the factors inverted. The last four numbers of page 62 might be extended considerably for oral drill.

63 A little assistance may have to be given to the children before they can work out the exercises as indicated. Do not hasten them with this work, and when any child finds an object too difficult to draw, let him use marks or dots. Chairs, horses, and children will probably be too difficult for children to draw, and any attempt to do it might divert their attention from the numerical operations.

64 Dwell with special emphasis upon the most difficult combinations in both multiplication and division. Frequent additions and subtractions by eights, beginning with 8 and 96, will help to fix the tables in the minds of the children.

65 These problems should be performed first by picturing as called for, and afterwards without aids of any kind.

66 See suggestions for page 64.

67 This is a review of what may profitably be done in various stages of the children's progress. Let the numbers of the table which are most difficult to learn be written in heavier lines than the others.

68–69 Review drill exercises for busy-work and for recitation.

70 To be performed without aids if possible. It may be necessary to give some assistance in **15**.

71–73 Weights and measures new to the children should be taught with the objects. The simpler problems also should be first taught with objects. Afterwards, pictures representing the weights and measures may be used. Squares or oblongs of approximately correct relative size would suffice for this purpose. For example, in **11**, a square whose edge is 1 in. could represent the bushel, marked off in four equal parts to represent pecks. Opposite the spaces the prices could be placed, and from them the required answers given. Problems involving the use of pounds and dozens also can be illustrated in the same way. The children if permitted will give some very interesting graphic solutions of these problems. When the children have a clear idea of the processes by the aid of objects and drawings, they should be expected to solve the problems promptly without aid of any kind.

74 When the children know thoroughly and can tell at sight all combinations to 100, using any number for the adding, subtracting, multiplying, or dividing number to 10, they are ready for the work of this section, which includes the addition and subtraction of any number to 100, and the multiplication and division of numbers to 100 by numbers to 20. To accomplish this it will be necessary to follow slowly the order indicated, and to give frequent reviews.

75–82 If the work is found too difficult at any point, observe the preceding steps. For example, if the children cannot perform readily the work on page 76, let them first add by tens and units separately; thus, in **10**: 30 and 40 are 70, and 9 are 79; 50 and 30 are 80, and 2 are 82, etc. After a rapid review of this kind they will be ready to analyze and add mentally. Subtraction will be found more difficult than addition, and more time should be spent upon it. The separation of the number to be subtracted into tens and units may be frequently necessary; for example, in **2**, page 81: 75 less 30 are 45 less 9 are 36, etc. A complete mastery of the subtraction of any number from 100 is especially desirable, since such subtraction will be found very convenient in making change. There is no more reason for adding coins piece by piece

in making change for a dollar than in making change for a dime. One good method of drill in subtraction is to place a number upon the blackboard, and give orally a number with the expectation that the children will name another number, which added to the given number will make the number on the board. For example, the teacher writes 89 on the board, and says: Forty-six. *Pupils.* Forty-three. *T.* Twenty-four. *P.* Sixty-five, etc. Rapid individual work of this kind will be found useful.

83–84 The previous work of this section is expected to be performed without the aid of figures. For variety, and for the purpose of keeping fresh in the children's minds the decimal system of notation, these written exercises are given. If the processes required are not fully understood, let two or three of the examples be performed with sticks, as was previously shown. Then let the children add, explaining the process of "carrying" as they add. The explanation may be very simple; thus, in **36**: 9 units and 4 units are 13 units, equal to 1 ten and 3 units. Write 3 units in the place of units. 1 ten and 5 tens and 3 tens are 9 tens. Write 9 tens in the place of tens. *Answer:* 9 tens and 3 units, or 93.

85–86 An almost unlimited number of examples may be given in connection with these tables. It will be observed that the numbers are arranged differently in the three tables, and that they can therefore be used at any time for a particular purpose. For example, if it is found that the combination $8 + 5$ in connection with large numbers is especially hard for the children to remember, drill may first be given in Table A, lines *s*, *p*, and *n*. Add 8 to numbers in line *p* as far as *i*. Add 18 as far as *g*. Add 38 as far as *f*. Do the same to numbers in *s*. Subtract 5, 15, etc., from line *n*. The same may next be done with Table B, lines *o*, *p*, and *n*.

If it is desired to drill with 14 as a subtrahend, the order of drill-work would be: In Table B, subtract 14 from numbers in columns *c*, *d*, *e*, etc. Subtract 14 from numbers in lines *l*, *m*, etc., beginning with *c*. In Table C, subtract 14 from numbers in

columns *c*, *d*, *e*, etc. Subtract 14 from numbers in line *l* beginning with *c*, from numbers in *m*, *n*, etc.

If it is desired to add or subtract a number from numbers whose tens are the same and whose units are unlike, the columns in tables B and C, could be used; and if it is desired to add or subtract a number from numbers whose tens are unlike and whose units are the same, the lines of A and B could be used. Other uses of the tables will appear for multiplication and division.

For methods of blackboard drill, see Manual, pages 33 and 34.

87–90 Ability to multiply numbers by any number to 10, and to divide by numbers to 20, would seem to be of sufficient convenience in everyday life to give some attention to these processes in the lower grades. The use of the drill tables on the two preceding pages may help to supplement these pages in giving the needed drill. Place upon the board other drill tables, consisting of the multiples to 100 of all numbers as far as 20, — one with the multiples set in order, and another with the multiples not in any regular order. Drill upon these until the multiples are readily recognized. There is no reason why 72 cannot be as quickly recognized to be a multiple of 18 as of 9.

91 Some attention to the form of answers to problems should be given, with the understanding always that a correct form in itself is not an end, but only a means of showing that the process of thinking is correct. First be sure that the child understands a process, and then lead him to express the steps of the process in correct language. This page of problems, and others which follow, are supposed to furnish models of simple explanations. To fix some of these forms in the mind, it might be well to give other numbers than those stated in a problem. For example, in **9**, after the problem is solved as given, the teacher might say, "What will four books cost?" or, "Suppose six books cost eighteen dollars, what would one book cost? two books?" etc. If the thought is not clearly expressed, do not correct the form of answers until it is clear that the process is understood. Objects or drawings should frequently be used, both in testing the child's knowledge of a process, and in teaching him the process.

92 Some of these problems may have to be illustrated before the statements of steps and answers are given, but let the children illustrate them if they can without assistance. With the illustrations before the children, the teacher may ask, as in **5**, "One apple costs what?" "Eight apples will cost how many times two cents?" Then the problem can be solved in the form given, to be followed by the use of other numbers. The statement of steps in short sentences, as in **7** and **8**, will be found useful and profitable as busy-work. If in any case the numbers seem too large, use smaller ones; as in **3**, the teacher may say, "What will it cost at that rate to get two collars washed? four collars? eight collars? four collars and two cuffs?" etc.

93 A little preliminary questioning or teaching may be advisable before some of these problems are given; for example, **3** and **6**. Problems similar to **4** will be found interesting and profitable. The processes involved in **8** and **9** will be readily understood if similar problems with smaller numbers are first given.

94 To learn once for all the number of days in each month is quite important, and need not be so difficult as many suppose. The verse, "Thirty days hath September," could be learned, but it would be better to learn the facts that four of the months have 30 days, that one has 28 in all years except leap years, and that all the rest have 31 days. Frequent applications of these facts in little problems will fix them in the mind so that they will not be forgotten. Children never tire of measuring, and the rod line which the children are directed to make, will be a never-ending source of pleasure and profit.

95 This work should be continued until the children have a tolerably accurate idea of short distances.

96 Reduction, first with the aid of a measure, and afterwards as far as possible with no aid, will be found an excellent preparation for subsequent work. Paper of different sizes should be counted out in sheets and quires, and some simple problems given with the paper in sight of the children.

97–104 Most of the *facts* contained in these problems should have been taught previously. If in any problem there is a fact or process quite new to the children, it should be taught objectively, the rule being observed that nothing should be told the child which he can find out for himself. The illustration of problems by the children is advised whenever the processes seem too difficult, but the same or similar problems should afterwards be performed without the aid of objects or drawings.

Blackboard Drill. — For the sake of variety, it may be well sometimes to give a drill in adding, subtracting, multiplying, and dividing from the blackboard. If it is found, for example, that the pupils are slow in adding and subtracting, put on the board columns beginning with only ones and twos, and increasing; new numbers to be added or subtracted gradually. Practice should continue until as great facility is had in adding and subtracting 7, 8, and 9, as in adding and subtracting 1, 2, and 3. The columns will appear as follows:

1	3	2	3	3	4	3	4	5	3	8	6	8	9
2	1	1	2	2	3	6	3	6	8	3	8	9	6
1	2	3	1	4	5	2	2	7	4	7	7	8	9
2	2	4	3	5	3	4	7	4	5	6	9	6	3
2	3	2	4	1	2	3	2	6	8	5	5	7	4
1	1	1	2	3	4	5	5	7	7	7	4	3	7
1	2	3	4	4	5	6	6	3	4	8	3	8	8
2	3	4	3	2	4	2	3	5	5	6	8	7	9
1	2	2	1	5	3	3	4	4	6	3	7	9	8
2	1	3	4	1	2	5	6	6	7	8	6	5	9
2	3	1	3	3	5	6	5	3	6	5	5	7	7
—	—	—	—	—	—	—	—	—	—	—	—	—	—

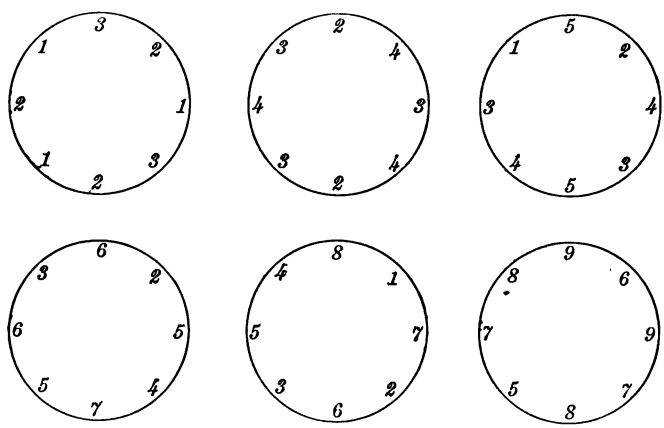
Entirely new combinations may be made by substituting a number in place of the first number of the column to be added, or by placing a new number below or above the column, and beginning with that number to add.

The same columns may be used for subtraction by placing any number above the column, from which successive subtractions are made. Thus, if 50 were placed above the first column, the children would be led to say, 49, 47, 46, 44, etc.

By the same method, correctness of work in addition may be proved. For example, if the pupils in adding the first column have an answer of 17, they can begin with that number and subtract successively the numbers of the column. If in subtracting the last number the remainder is nothing, the answer is presumed to be correct.

For drill in multiplication and division the same columns may be used, the number to be multiplied or divided being placed above the column.

The same progressive drill-work may be given by placing the numbers in a circle, as follows:



The pupils could add in either direction to any given number, or from any number placed in the centre, subtractions to zero could be made. Numbers could also be placed in the centre for multiplication and division.

SECTION V.

NOTES FOR BOOK NUMBER THREE.

By far the greater part of the work laid out in this book is with numbers under 1000, and if a year is to be given to the book, at least seven months may well be spent upon this part of it. Thoroughness in the use of small numbers means a saving of time and increased power to apply what is learned in the higher grades.

The apparatus needed for teaching and illustrating these exercises is a large number of short wooden sticks with rubber bands, and all the common weights and measures, such as the foot rule, yard stick, rod line, gill, quart, gallon, peck, bushel, and balance for weighing.

Read carefully the Note to Teachers on pages iii and iv, particularly what is said in the paragraph marked 2.

1–11 These exercises are a partial review of what has been given in Book II. Pupils should be able to perform them with facility before Section II. is begun. If for any reason there is difficulty in performing or in understanding any of the exercises, they should be taught and illustrated as previously recommended. If the difficulty lies in the computations involved in the four fundamental rules, drill is needed until the difficulty is removed. For a good method of drill upon simple computations to 100 see Book II., pages 85 and 86; also Manual, pages 33 and 34.

If there is difficulty in making the applications, the use of small numbers and a graphic representation of the processes by drawings are advised.

12–14 The new grouping of hundreds to make the thousand should be taught by objects as before, and if the objects have previously been sticks in bundles of ten, the same objects should now be used. First review what has been taught, — 10 units or ones of sticks to make a ten, and 10 tens of sticks to make a

hundred. Let the pupils work with these tens and units until their use is quite familiar. Lead the pupils to put into bundles and represent 2, 3, 4, etc., hundreds, varying the order of numbering and of representing. For example, the teacher puts before the pupils piles of single sticks and bundles of tens, saying, "Show me 6 tens. How many units in 6 tens? Who can write that number on the board? Show me seventy-six; eighty-three; sixty-four; ninety-seven. Show me 10 tens. What is that called? How is one hundred written? Why ciphers in the units and tens places? We will put the 10 tens together in a bundle. How many hundreds in this bundle? Show me 2 hundreds. These together are called two hundred. How shall we write the number two hundred? Show me 3 hundred; 4 hundred. Who will write four hundred on the board? Show me 1 hundred and 2 tens. 2 tens is called what? What then is the whole number called? How will you write 1 hundred and 2 tens, or one hundred twenty? Show me 2 hundreds and 3 tens. What shall we call this number? How shall we write it?" In this way the teacher goes on teaching the hundreds, then the hundreds and tens, and finally the hundreds, tens, and units, until the thousand is reached. Now comes the drill begun on page 12. The exercises given on these pages indicate the order which may be followed, but they may not be sufficient in number for some pupils.

Teach the term *sum*, and lead the pupils to use it.

15–16 These exercises are supposed to be performed with and without the aid of objects and without figures. If in any case the answers cannot be readily given, let the intermediate steps be given. For example, in **11**, page 15, let the pupils say: 660, 760, 790; 380, 580, 630; and in **7**, page 16: 850, 650, 490; 720, 420, 380.

17–18 These exercises should be performed on paper or slate, first with sticks and afterwards without sticks. Lead the pupils to put together the sticks in each case *before* the result is written; thus, in **19**, page 17, the pupils should first put in columns the tens and units of sticks to be added, and write the corresponding

numbers on paper or slate. They then put together the 3, 4, and 4 sticks. 10 of these they should put into a ten bundle and say, "3 units and 4 units and 4 units are 11 units, equal to 1 ten and 1 unit. I write the 1 unit in the place of units, and add the 1 ten to the tens." Then putting the bundles of tens together they say, "1 ten and 4 tens and 2 tens and 3 tens are 10 tens, equal to 1 hundred and 0 tens. I write the 1 hundred and 0 tens in their places, and have for an answer 101."

When a sufficient amount of drill with the sticks has been given, the pupils should place in columns the numbers beginning with **3**, page 17, and proceed in order until all exercises on these pages are performed.

It is permitted sometimes for convenience in "proving" answers to write the "carrying" figure above the column in which the addition is made; but it is not well for pupils, either in addition or in multiplication, to depend upon writing down the carrying figure; thus, **37**, page 17:

11
314
151
268
733

A good way of preserving the carrying figure for proof, is to write the result of each addition, thus:

314
151
268
13
12
6
733

This is found especially useful in adding long columns.

Do not insist upon formal and elaborate "explanations." It is enough for the pupils to express what they recall of the process of uniting together ten of one denomination to make one of the next higher. For example, in **1**, page 18, the pupils might be led to say: "2 units and 7 units and 6 units are 15 units, equal to 1 ten and 5 units. I write the 5 units and add the 1 ten with the tens. 1 and 4 and 1 and 8 tens are 14 tens, equal to 1 hundred and 4 tens. I write the 4 tens and add the 1 hundred with the hundreds. 1 and 1 and 2 and 3 hundreds are 7 hundreds, which I write. *Answer:* 745." Of course this form or any set form should not be

put before them as a model. What is desired of them is simply to recall impressions which were made in the objective work, and to express what they think.

19 *Answers:* **1** 707. **2** 861. **3** 718. **4** 841. **5** 690. **6** 781. **7** 765. **8** 807. **9** 795. **10** 998. **11** 867. **12** 835. **13** 971. **14** 840. **15** 815. **16** 854.

A large number of drill examples may be made by asking the pupils to place given numbers above or below in the columns, or the teacher could dictate to the pupils the numbers here given and one other number. The answers could be found by adding the extra number to the answers above given. In adding aloud, permit only results to be given, and lead the children to add by pairs. If difficulty is found in this, put pairs of numbers on the board for them to add quickly at sight as follows: $\begin{matrix}8\\6\end{matrix}$ $\begin{matrix}7\\19\end{matrix}$ $\begin{matrix}13\\14\end{matrix}$ etc. It will not be difficult after some practice to add in the manner desired; thus, in **1**, the answers should be 15, 26, 39, 46, etc.

20–21 These exercises should be performed first with objects, the pupils being led to use the sticks as indicated before the results are written; thus, in **12**, page 20, the pupils place before them 6 bundles of hundreds and 7 bundles of tens, and after writing on paper or slate 670 above 286, say, "I wish to take 286 from 670." Having no units they take a ten-bundle, and after separating into units and taking from them 6 sticks, say, "6 units from 10 units are 4 units. I write 4 in the units place. I took 1 ten from the 7 tens, and there are left 6 tens. I cannot take 8 tens from 6 tens, so I take one of the hundreds." This he does, and after resolving it into tens and placing them with the 6 tens, says, "1 hundred equals 10 tens. 10 tens and 6 tens equal 16 tens." He then goes on with the subtraction, writing after each result is obtained, and telling what is done.

As in addition, the statements should be a simple and natural expression of the pupil's thought.

At this point teach the terms *minuend*, *subtrahend*, and *remainder*, and lead the pupils to use them in reading the exercises and answers.

22 *Answers:* **1** 591. **2** 861. **3** 961. **4** 748. **5** 976. **6** 960. **7** 717. **8** 893. **9** 903. **10** 752. **11** 957. **12** 979. **13** 759. **14** 1027. **15** 866. **16** 857. **17** 988. **18** 778. **19** 714 495 603. **20** 307 350 159. **21** 270.

This drill table may be used in supplying a large number of examples in addition and subtraction. For example, the pupils may be told to add *ba* from *A* to *K* or from *D* to *P*, or the same work may be dictated to the pupils. Other columns as *cb*, *dc*, etc., could be given in the same way. Practice also in adding by lines could be given from the table. Other columns than those given for subtraction could be given as *dcb*, *edc*, etc.

23 *Answers:* **1** 509. **2** 216 492 307 139 219. **3** 176 317 252 517 187 186. **4** 292 704 237 179 178 124. **5** 616 332 174 203 288 381. **6** 206 337 186 291 144 69. **7** 278 184 289 249 336 186. **8** 303 163 226 340 203 197. **9** 667 583 83 387 626 287. **10** 963 cd. **11** 217 sheep. **12** 625 hours. **13** 71 highest grades 176 lowest grades.

24 *Answers:* **1** 226 194 292 393 398. **2** 643 188 419 361 206. **3** 60 83 294 147 139. **4** 39 280 361 335 228. **5** 348 266 719 304 422. **6** 593 670 206 606 445 101. **7** 157. **8** 268. **9** 508. **10** 319. **11** 342. **12** 287. **13** 306. **14** 302. **15** 305. **16** 276. **17** 321. **18** 244. **19** 290. **20** 372. **21** 612. **22** 426. **23** 104 72. **24** 124 120. **25** 164 82.

From this table a large amount of drill-work may be given, the pupils taking different combinations.

25 *Answers:* **1** 873 bu. corn 1026 bu. wheat. **2** 388 pear trees. **3** 742 mi. **4** 195 ft. **5** 930 papers. **6** 674 papers. **8** 284 yrs. 1492, Dis. 1620, Pilgrims 1776, Dec. of Independence.

Some of these problems suggest a kind of work that may be given from statistics found in the latest almanac.

26 *Answers:* **1** 365 d. **2** $396. **3** $65. **4** 837 lb. **5** 1091. **6** 350 mi.

The making of original problems is good practice in language. Useful local statistics may be used profitably for this purpose.

27–29 Observe the order of work here required, and do not abandon the objects because pupils can find the correct answers without them.

30 *Answers:* **1** 99. **2** 884. **3** 604. **4** 424. **5** 624. **6** 798. **7** 872. **8** 590. **9** 678. **10** 720. **11** 780. **12** 840. **13** 805. **14** 905. **15** 928. **16** 815. **17** 748. **18** 984. **19** 960. **20** 772. **21** 976. **22** 819. **23** 776. **24** 897. **25** 936. **26** 1088. **27** 385. **28** 978. **29** 621. **30** 820. **31** 915. **32** 988. **33** 588. **34** 774. **35** 1050. **36** 995. **37** 861. **38** 880. **39** 872. **40** 948. **41** 957. **42** 804. **43** 649. **44** 963. **45** 1062. **46** 1188. **47** 1228. **48** 567. **49** 910.

Let work with objects accompany the description of the process and precede the writing of results; thus, in **17**, the pupils should have four groups of sticks to put together, each group to have 1 hundred-bundle, 8 ten-bundles, and 7 units. In putting together the single sticks or units the pupils should say: "4 times 7 units are 28 units"; and then in putting the twenty into two bundles of tens should add: "28 units equal 2 tens and 8 units. I write the 8 units in the place of units, and add the 2 tens to the next product." Then putting together the tens the pupils say: "4 times 8 tens are 32 tens plus 2 tens are 34 tens," etc.

Teach the terms *multiplicand*, *multiplier*, and *product*, and lead the pupils to use the terms in reading and explaining problems.

31 *Answers:* **1** *a* 1044; *b* 1216; *c* 768; *d* 984; *e* 936. **2** *a* 406; *b* 672; *c* 525; *d* 574; *e* 679. **3** *a* 783; *b* 585; *c* 828; *d* 342; *e* 657. **4** *a* 1296; *b* 1496; *c* 864; *d* 1016; *e* 1472. **5** *a* 616; *b* 528; *c* 836; *d* 352; *e* 737. **6** *a* 445 534 623 712 801 890; *b* 380 456 532 608 684 760; *c* 265 318 371 424 477 530; *d* 460 552 644 736 828

920. **7** *a* 130 156 182 208 234 260; *b* 445 534 623 712 801 890; *c* 385 462 539 616 693 770; *d* 465 558 657 744 837 930. **8** *a* 410 492 574 656 738 820; *b* 465 558 657 744 837 930; *c* 335 402 469 536 603 670; *d* 425 510 595 680 765 850. **9** *a* 385 462 539 616 693 770; *b* 465 558 657 744 837 930; *c* 430 516 602 688 774 860; *d* 135 162 189 216 243 270. **10** *a* 380 456 532 608 684 760; *b* 240 288 336 384 432 480; *c* 460 552 644 736 828 920; *d* 380 456 532 608 684 760. **11** *a* 385 462 539 616 693 770; *b* 340 408 476 544 612 680; *c* 285 342 399 456 513 570; *d* 210 252 294 336 378 420. **12** *a* 445 534 623 712 801 890; *b* 380 456 532 608 684 760; *c* 465 558 657 744 837 930; *d* 420 504 588 672 756 840. **13** 620 570 654 566 876 732 1026. **14** *a* 636; *b* 768; *c* 990; *d* 1092; *e* 1134; *f* 1182. **15** *a* 609; *b* 672; *c* 518; *d* 266; *e* 574; *f* 679. **16** *a* 448; *b* 384; *c* 576; *d* 744; *e* 520; *f* 768. **17** *a* 513; *b* 783; *c* 324; *d* 738; *e* 882; *f* 774. **18** *a* 539; *b* 726; *c* 913; *d* 836; *e* 638; *f* 737. **19** 70 41 51.

32 *Answers:* **1** *a* 787; *b* 808; *c* 1064. **2** *a* 964; *b* 1078; *c* 1023. **3** *a* 992; *b* 1067; *c* 988. **4** *a* 288; *b* 207; *c* 463. **5** *a* 150; *b* 501; *c* 688. **6** *a* 118; *b* 146; *c* 574.

33 The exercises of this page should be performed with and without the use of objects. The statement of steps and process given should be the same in one case as in the other.

34 *Answers:* **1** 736. **2** 676. **3** 594. **4** 852. **5** 812. **6** 645. **7** 525. **8** 946. **9** 720. **10** 756. **11** 546. **12** 931. **13** 782. **14** 702. **15** 528. **16** 432. **17** 1157. **18** 1358. **19** 646. **20** 1288. **21** 1548. **22** 931. **23** 885. **24** 741. **25** 1472. **26** 1144. **27** 425. **28** 1148. **29** 836. **30** 1170. **31** 1066. **32** 688. **33** 476. **34** 1022. **35** 840. **36** *a* 1064; *b* 700; *c* 1022; *d* 728; *e* 1204; *f* 1008. **37** *a* 795; *b* 915;

c 1125; *d* 1035; *e* 855; *f* 570. **38** *a* 464; *b* 592; *c* 832; *d* 624; *e* 720; *f* 288. **39** *a* 663; *b* 765; *c* 323; *d* 408; *e* 527; *f* 289. **40** 1246 1035 1343. **41** 1314 1273 910.

35 *Answers:* **1** 1134 300 235 1186. **2** 623 21 254 343. **3** 552. **4** 884. **5** 837. **6** 782. **7** 770. **8** 768. **9** 902. **10** 864. **11** 918. **12** 903. **13** 870. **14** 832. **15** 980. **16** 836. **17** 936. **18** 848. **19** 806. **20** 870. **21** 966. **22** 621. **23** 638. **24** 700. **25** 1054. **26** 970. **27** 1073. **28** 840. **29** 882. **30** 864.

These should be performed without objects, the pupils giving in clear form statements of process.

The objective work in division of large numbers here begun should be very careful and methodical. In **33** another form of statement would be: "$\frac{1}{4}$ of 8 tens of sticks is how many? $\frac{1}{4}$ of 4 sticks is how many? $\frac{1}{4}$ of 84 sticks is how many?" This process is sometimes called "*partition*" to distinguish it from "*division*," or the process of finding how many times one number is contained in another number.

36 Lead the pupils to use the objects and to make statements as they divide. The form of questions used for exercises on page 35 may be changed to asking how many times one number will be contained in another number. For example, in **1**: "How many times are 2 sticks contained in 200 sticks? How many times are 2 sticks contained in 20 sticks? How many times are 2 sticks contained in 220 sticks?"

If multiplication has been taught by objects thoroughly, it will not be necessary to continue with the objects in division to a great extent. It may be assumed, for example in **6**, that the pupils know that 2 is contained in 16 tens 8 tens times, and all similar examples may be so explained without the aid of objects. In **8** *c* a new step is taken, and should be taught with objects thus: "2 sticks are contained in 13 tens of sticks how many tens times, and how many tens remainder? 2 sticks are contained in 1 ten or 10 how many times? What is the answer?" This principle is

further applied in **11**, and a similar course of teaching should be taken.

After the examples of this page have been performed in the way indicated, the same examples may be performed as if sticks and not times were called for. This is a more simple process to teach. For example, in **1**, the teacher should lead the pupils to get ½ of 2 hundreds, then ½ of 2 tens, thus finding ½ of 220 to be 1 hundred and 1 ten or 110. A rapid oral review without objects should be given before the next page is taken.

37 This page of exercises may be taken, without objects, orally. In such exercises as **23** *a*, the pupils may be led to say first: "4 in 120, 30 times. 4 in 132, 33 times." Afterwards they may perform the exercises without analysis.

38 The same course with objects is to be pursued here as was advised for the exercises on page 36, with the added feature of expressing with figures the results as they are found. Teach the terms *dividend*, *divisor*, and *quotient*, and lead the pupils to use them.

39 Long division, if thought best, may be begun with small numbers, and may be performed without objects. A good form of long division is to place the divisor at the left of the dividend, and the quotient above the dividend, as shown below.

In teaching long division, the pupils should be led to see that there are no new processes to learn, but that it is the same as short division, with the products written out instead of carried in the mind. The following order of teaching is suggested, it being understood that as the pupils answer the questions the numbers are written.

```
  49
17)833
  68
 153
 153
```

2 is contained in 4 tens how many tens times? 17 is contained in 83 tens how many tens times? How shall we find the remainder that is not divided by 17? Multiplying 17 by 4 tens and subtracting, what have we? This number of tens and 3 units is what number? 17 is contained in 153 how many times? 17 multiplied by 9 is what? What remainder is there? What is the answer?

The teaching of partition in which the fractional part of a number is found is also to be taught, although the form of work is the same. The statement of steps taken in long division should be at first very simple, following somewhat closely the order of teaching as given above.

40 *Answers:* **1** $22\frac{1}{2}$. **2** $14\frac{3}{4}$. **3** $21\frac{1}{3}$. **4** $20\frac{1}{3}$. **5** $21\frac{1}{2}$. **6** $20\frac{1}{4}$. **7** $20\frac{2}{3}$. **8** $20\frac{2}{3}$. **9** $20\frac{2}{3}$. **10** $21\frac{9}{37}$. **11** $21\frac{1}{2}$. **12** $20\frac{2}{3}$. **13** $21\frac{7}{37}$. **14** $22\frac{1}{3}$. **15** $21\frac{1}{3}$. **16** $22\frac{2}{4}$. **17** $20\frac{1}{4}$. **18** $21\frac{3}{4}$. **19** $19\frac{2}{4}$. **20** $21\frac{1}{4}$. **21** $23\frac{3}{3}$. **22** $22\frac{2}{4}$. **23** $16\frac{2}{3}$. **24** $16\frac{7}{4}$. **25** $26\frac{1}{3}$. **26** $14\frac{4}{6}$. **27** $20\frac{3}{4}$. **28** $16\frac{1}{3}$. **29** $19\frac{2}{3}$. **30** $25\frac{2}{3}$. **31** *a* $16\frac{7}{18}$; *b* $25\frac{4}{18}$; *c* $43\frac{9}{18}$; *d* $50\frac{4}{18}$. **32** *a* $23\frac{5}{17}$; *b* $42\frac{5}{17}$; *c* $29\frac{7}{17}$; *d* $47\frac{4}{17}$. **33** *a* $62\frac{2}{14}$; *b* $49\frac{6}{14}$; *c* $50\frac{7}{14}$; *d* $21\frac{1}{14}$. **34** *a* $11\frac{1}{3}$; *b* $10\frac{2}{3}$; *c* $8\frac{2}{3}$; *d* $10\frac{2}{3}$. **35** 76 $24\frac{7}{18}$. **36** $50\frac{2}{14}$ $52\frac{4}{16}$. **37** $56\frac{1}{17}$ $31\frac{1}{13}$. **38** $41\frac{9}{11}$ $46\frac{5}{15}$. **39** 126. **40** $62\frac{1}{4}$. **41** $40\frac{5}{8}$. **42** $25\frac{4}{8}$. **43** 18. **44** 15. **45** 73. **46** $49\frac{1}{3}$. **47** $36\frac{1}{7}$. **48** $27\frac{7}{9}$. **49** $25\frac{2}{10}$. **50** 27.

Lead the pupils to divide in some cases by a trial divisor, consisting of the first figure of the divisor, to see about how many times the divisor is contained in the dividend. This will be more helpful when larger numbers are used.

41 *Answers:* **1** $28\frac{5}{8}$. **2** $51\frac{1}{2}$. **3** $78\frac{2}{4}$. **4** $73\frac{4}{8}$. **5** 294. **6** 105. **7** $115\frac{2}{3}$. **8** $123\frac{2}{3}$. **9** $115\frac{1}{7}$. **10** $63\frac{1}{17}$. **11** 68. **12** 112. **13** $113\frac{6}{7}$. **14** $67\frac{8}{10}$. **15** $134\frac{3}{4}$. **16** 442. **17** $50\frac{8}{12}$. **18** $135\frac{1}{7}$.

Exercises **19** to **53** are for drill, either in recitation or in study. For the sake of accuracy and of securing practice in multiplication, it is well to have their work in these exercises proved in the usual way, — multiplying the quotient by the divisor and adding the remainder to the amount.

42 Some review work which the pupils should be able to do without help of any kind. Let the statement of steps and answers be a simple and natural expression of the pupils' thought; thus, in **12**: "It will take as many days to burn 35 cords as there are

sevens in 35"; or, "It will take as many days as 7 cords is contained in 35 cords"; or, "To find how many days, I divide 35 cords by 7 cords."

43 Lead the pupils into the habit of writing out the steps of a problem in good form, and also of marking the denomination of each number. Two forms may be required, — one form in which the steps are only indicated, and another form in which all the steps are worked out; thus, in **1**:

$\frac{1}{18}$ of 396 mi. = average number of mi. in 1 da.

18) 396 mi. in 18 da.
22 mi. in 1 da.

44 Extend the work in making and solving original problems. It will be found good work for review.

45 Some teaching of the writing of numbers in U. S. money should precede these exercises. Show how two ounces, eight pounds, six dollars, may be expressed by figures and words, and also by figures and abbreviations. Give the sign for dollars, and show its use by examples. Lead the pupils to see that a number of cents less than one hundred is written at the right of the point, the number of dimes expressed by the first figure at the right of the point, and the number of cents less than ten expressed by the second figure. After a little of such work, the pupils ought to be ready for the exercises on this page.

46 If the pupils have a thorough knowledge of writing numbers in U. S. money they should have little difficulty in adding, subtracting, multiplying, and dividing such numbers. Before the exercises of this page are given it would be well to have some simple exercises in pointing off, such as the following: "In 200 cents how many dollars? Where shall I place the point to show the number of dollars? Point off the dollars in 300 cents; 220 cents; 640 cents; 104 cents. How should these numbers be written for addition and subtraction?" In multiplication, lead the pupils first to multiply dollars, then cents, and finally dollars and cents, such as the following: \$84 × 4; \$61 × 8; \$75 × 6; \$0.80 × 2;

0.75×4; 1.50×6; 2.24×3; 1.08×4; 1.38×6. In this work, insist upon the point being made in all cases, and lead the children to see that, if the multiplicand is cents the product will be cents, and that it must be pointed off accordingly.

It is well in the written statement of the solution of problems to write what is given and what is required to be found; thus, in **1**:

Given the cost of 1 cow and 1 horse.

To find the cost of 6 cows and 3 horses.

$64	cost of 1 cow.	$140	cost of 1 horse.
6		3	
$384	" " 6 cows.	$420	" " 3 horses.
420	" " 3 horses.		
$804	" " 6 cows and 3 horses.		

47 Before giving **5**, **6**, **7**, and **8** to the pupils, lead them to see that in all such problems the dividend and divisor must be of the same denomination. This may be done by showing that 2 cents is contained in 6 dimes or 6 dollars more than 3 times. Give a few exercises like the following: "3 cents in 3 dimes how many times? 6 cents in 3 dimes? 2 dimes in 1 dollar? 4 dimes in 2 dollars? 5 cents in 2 dollars? 4 cents in 1 dollar? 4 cents in 8 dollars?" **9** and **10** should be performed and explained by partition, in which the answer is the same denomination as the sum divided; thus: $\frac{1}{2}$ of $4 = $2; $\frac{1}{5}$ of $8.25 = $1.65. In these and similar exercises pay particular attention to pointing off. In exercises like the last of **10** it may be well to reduce the sum divided to cents before dividing; thus: "$1\frac{1}{2}$ of 408 cents is 34 cents."

48 Let the pupils perform as many of these orally as they can, **1**, **2**, and **4** to be done by partition, and **3** by division. A simple statement of process should be made to represent the difference between these two operations; thus, in **1** *a*: "1 qt. of kerosene will cost $\frac{1}{4}$ as much as 4 qt. $\frac{1}{4}$ of $36¢ = 9¢$." And **3**: "Since 1 book can be bought for $25¢$, as many books can be bought for $4.25 as there are twenty-fives in 425, or 17 books"; or, "as many books as $25¢$ is contained times in $425¢$."

If the use of fractions, such as are given in **6** and **7**, is not familiar to the pupils, give some simple work with blocks, as $\frac{1}{4}$ of 16 blocks, $\frac{3}{4}$ of 16 blocks, $\frac{1}{5}$ of 20 blocks. Discs or drawings of circles, squares, or lines may be used in finding the fractional part of numbers that cannot be evenly found; thus: $\frac{1}{3}$ of 4 circles, $\frac{2}{3}$ of 4 circles, $\frac{1}{3}$ of 6 circles, $\frac{2}{3}$ of 6 circles, etc., may be taught.

49–51 No new principle is involved in these exercises. Statements like the following may be placed sometimes before problems, so as to make their conditions more clear to the pupils, thus:

36 oranges cost 72¢	1 yd. costs 14¢
1 orange costs ?	? " cost 224¢

It is well occasionally to ask such questions as: "8 lb. will cost how many times as much as 1 lb.? how many times as much as 2 lb.? how many times as much as 4 lb.? What part as much as 16 lb.?" Pupils should be able to recognize from the first that a fractional part of the given number is taken when the required answer is of the same denomination as the number divided, and that division of one number by another is performed when the number of parts is required.

52 Great care should be taken in making these bills, directions being given as to ruling, date, receipt, etc. The pupils should learn to make their own rulings.

53 Toy money may be first used in performing these exercises, if necessary. Pupils should be able to tell instantly the difference between one dollar and any sum of money below that amount, thus avoiding the necessity of adding in making change.

54 As many of the problems in this section as possible should be performed orally. Some of them may be written out for the sake of practice in written analysis. The making of original problems may be considerably extended.

55–64 Weights and measures should be used in performing the problems of this section when needed. It will be necessary to use them considerably if they have not been used before; and if they have been used before, the pupils should be led to work

with the measures or drawings of the measures whenever the conditions of a problem are not clearly understood. Nearly all of these problems should be performed without figures; but for practice in stating the steps of the solution, some of them may be written out in full. It is advisable for the teachers to spend some time in showing the pupils a good form of solution. The following written solutions are suggested:

Exercise 10, page 58.

Given the number of bushels of grain on hand.

To find the number of bushels more to fill an order.

1000 bu. of grain ordered.
735 " " " on hand.
265 " " " required to fill the order.

Exercise 6, page 62.

Given the amount of paper a bookseller had and sold.

To find the amount he had left.

10 reams 12 quires = 212 quires, amt. of paper on hand.
8 " 4 " = 164 " " " " sold.
48 " " " " remaining.

If thought best, the parts of the solution may be ruled off in the following manner:

Exercise 13, page 63.

Required.	Solution.	Answer.
Cost of Sugar.	30 lb. in 1 tub. 28 " " 1 " 58 " " 2 tubs. $0.13 cost of 1 lb. × 58 104 65 $7.54 cost of 58 lb.	$7.54

65–67 The pupils have had some practice in measuring, but it should not be presumed that they do not need more practice of the same kind. In addition to the writing out of steps in the solution of problems, the pupils should be led sometimes to draw a diagram representing the distances given and required. This should be done in such problems as **4**, **6**, **8**, **10**, **11**, **12**, and **16**, page 67.

68–70 Do not permit pupils at this stage to find the square contents of a rectangle by multiplying the length by the width. Lead them always to think of the number of units in a row to be multiplied by the number of rows. In the exercises on page 69 the required measurements should be made, but the fraction of an inch or foot may be omitted. Wherever it can be done, a plan of the surface should be drawn to scale. This should be done in nearly all of the problems on page 70. Oral and written statements of processes should be required. For example, in **7**, page 70, the oral statement might be: "In the passage-way there are 10 rows of blocks and 12 blocks in a row. Since in 1 row there are 12 blocks, in 10 rows there are 10 times 12 blocks, or 120 blocks."

71–73 If numbers to 1000, including the four fundamental rules, have been thoroughly taught by objects, the knowledge thus gained will serve as a foundation for the teaching of numbers in the higher orders. At this point the pupils are supposed to know that ten of any order make one of the next higher order, and that the value of the number expressed is determined by the position that the figure has with reference to the decimal point. Proceeding upon this basis, the pupils count the thousands precisely as the units were counted, until a thousand thousand is reached, when the number is called one million. The period of thousands, both in reading and writing, should be treated exactly as the period of units was treated, ten of each order making one of the next higher. Extend the work here given if necessary.

74 *Answers:* **9** 7703. **10** 15044. **11** 17788. **12** 11125. **13** 13888. **14** 18054. **15** 20089. **16** 20958. **17** 16987. **18** 13587. **19** 25294. **20** 11896.

The combinations in thousands will be found easy, if the corresponding combinations in the period of units are well understood. The first eight exercises can doubtless be performed mentally by the pupils, but for practice in expression the numbers should be written in full. For convenience the fifth and sixth orders should be named ten-thousand and hundred-thousand. The reason for carrying one for every ten of a lower order should be given in the same way as was taught for numbers below 1000. For example, in **15**, the pupil is led to say: "8 and 3 and 9 hundreds is 20 hundreds, equal to 2 thousands, to be added with the thousands. 2, 8, 12, 20 thousands, expressed as 2 ten-thousand, and 0 thousand," etc.

75 *Answers:* **1** 10421. **2** 15749. **3** 19378. **4** 14574. **5** 8003. **6** 15750. **7** 233269. **8** 93533. **9** 54877. **10** 110981. **11** 51720. **12** 674673. **13** 733299. **14** 884770. **15** 765277. **16** 67688. **17** 68185. **18** 155813. **19** 15403. **20** 16042. **21** 9558. **22** 10688.

Let the statement of steps in adding be continued until the pupils have a clear idea of the process. Give frequent exercises in dictating numbers for addition. This is best for seat-work, so as to be sure that all work independently.

76 *Answers:* **1** 218495. **2** 11084. **3** *a* 12734; *b* 16404; *c* 21729; *d* 15131; *e* 25809; *f* 20045; *g* 22148; *h* 25484; *i* 35362; *j* 14794; *k* 17256; *l* 18801; *m* 22300. **4** 96513. **5** 549. **6** 1378. **7** 2321. **8** 1047. **9** 7742. **10** 6942. **11** 2290 3144 1480 1930. **12** 3250 2173 3760 3236. **13** 2417 751 8690 86 6048 9099 3215. **14** 2468 3239 8189 5686 7005 9012 1021. **15** 267 820 8176 575 7058 4639 4423. **16** 8392 6401 8355 672 256 4165 312. **17** 3685 517 254 833 1158 48. **18** 3046 5995 1045 8401 6355 715. **19** 4915 3289 786 8387 704 1653. **20** 1007 5127 7564 69 859 6444.

Lead the pupils to give a statement of steps in the process of subtraction until it is clearly understood. If the process is not understood, use objects in the subtraction of numbers to 1000.

77 *Answers:* **1** 1682. **2** 5343. **3** 652. **4** 642. **5** 604. **6** 3264. **7** 5377. **8** 7077. **9** 6398. **10** 235. **11** 3237. **12** 2353. **13** 2318. **14** 2161. **15** 5665. **16** 5659. **17** 294. **18** 5659. **19** 969. **20** 8449. **21** 11456. **22** 12122. **23** 26808. **24** 28539. **25** 23999. **26** 6588. **27** 23175. **28** 3179. **29** 181044. **30** 273706. **31** 93341. **32** 588680. **33** 175629. **34** 302357. **35** 301245. **36** 176586.

Let the pupils prove their answers. Show, by use of small numbers, why the sum of the subtrahend and remainder ought to be the same as the minuend.

78 *Answers:* **1** *a* 633; *b* 203; *c* 25; *d* 1225; *e* 192; *f* 873; *g* 6566; *h* 92; *i* 5275; *j* 2569; *k* 666. **2** *a* 138; *b* 3002; *c* 25; *d* 244; *e* 733; *f* 2587; *g* 1316; *h* 91; *i* 929; *j* 609; *k* 177. **3** *a* 2445; *b* 252; *c* 6342; *d* 307; *e* 86; *f* 579; *g* 14; *h* 552; *i* 2239; *j* 1326; *k* 212. **4** 81208. **5** 25156. **6** 7992. **7** 8925. **8** 77611. **9** 92316. **10** 11102. **11** 6966. **12** 17212. **13** 10628.

The proof by subtraction is only given for practice in subtraction. The best proof in addition is made by adding columns down if they have been added up.

79 *Answers:* **1** *a* 25483; *b* 7233; *c* 20319; *d* 22857; *e* 7866; *f* 38134; *g* 21684; *h* 14280; *i* 8098; *j* 1582. **2** *f* 4074; *g* 7123; *h* 5003; *i* 2110; *j* 20. **3** *f* 2229; *g* 5175; *h* 3922; *i* 1618; *j* 122. **4** *f* 1774; *g* 435; *h* 1046; *i* 994; *j* 381. **5** *f* 5185; *g* 5269; *h* 2287; *i* 2249; *j* 1. **6** *a* 2066; *b* 5115; *c* 2169; *d* 3608; *e* 3592. **7** *a* 2509; *b* 289; *c* 1542; *d* 3023; *e* 41. **8** *a* 2912; *b* 119; *c* 2423; *d* 383; *e* 345. **9** *a* 2282; *b* 192; *c* 1688; *d* 2301; *e* 53. **10** 2918 mi. **11** 52768. **12** 30961. **13** 164932.

80 After these exercises have been performed orally, it may be well to express the work of some of them in figures. Such exercise will prepare the pupils for subsequent work.

81 *Answers:* **1** 3474. **2** 12274. **3** 20032. **4** 102848.
5 816458. **6** 28452. **7** 81492. **8** 94542. **9** 112208.
10 753354. **11** 37536. **12** 82768. **13** 102816. **14** 57312.
15 42840. **16** 45474. **17** 27870. **18** 127224. **19** 39702.
20 44520. **21** 11572. **22** 18972. **23** 17955. **24** 17088.
25 14382. **26** 24783. **27** 26828. **28** 36868. **29** 11025.
30 25826. **31** 48802. **32** 28006. **33** 34941. **34** 33579.
35 55476. **36** 21924. **37** 66720. **38** 46315. **39** 38016.
40 66690. **41** 49068. **42** 23374. **43** 76665. **44** 68992.

Before multiplying by tens and units as given in the exercises beginning with **12**, let the pupils have some review in multiplying smaller numbers with objects, to show that the right-hand figure in multiplying by tens is in the tens' column.

82 *Answers:* **1** 4760. **2** 2140. **3** 30000. **4** 86850.
5 19300. **6** 29940. **7** 5160. **8** 80480. **9** 32100.
10 \$1008.25. **11** \$4191.50. **12** \$85.68. **13** \$1402.56.
14 \$2.88. **15** \$9011.25. **16** \$1083.06. **17** \$4567.50.
18 \$4148.16. **19** \$5984.94. **20** \$506. **21** \$92.64.
22 \$137.19. **23** \$48.90. **24** *a* \$4889.40 \$7225.35 \$6575.46 \$8131.02 \$7503.75 \$6987.84 \$5688.06 \$6285.75 \$6092.61; *b* \$2135.60 \$3155.90 \$2872.04 \$3551.48 \$3277.50 \$3052.16 \$2484.44 \$2745.50 \$2661.14; *c* \$5295.20 \$7972.80 \$7255.68 \$8972.16 \$8280 \$7710.72 \$6276.48 \$6836 \$6722.88; *d* \$2529 \$3737.25 \$3401.10 \$4205.70 \$3881.25 \$3614.40 \$2942.10 \$3251.25 \$3151.35; *e* \$3867.80 \$5730.45 \$5215.02 \$6448.74 \$5951.25 \$5542.08 \$4511.22 \$4985.25 \$4832.07; *f* \$4158.80 \$6145.70 \$5592.92 \$6916.04 \$6382.50 \$5943.68 \$4838.12 \$5346.50 \$5182.22; *g* \$3305.80 \$4899.95 \$4459.22 \$5514.14 \$5088.75 \$4738.88 \$3857.42 \$4252.75 \$4131.77.
25 \$228. **26** \$10560. **27** \$231904.

83 Some of these exercises should be performed by the aid of figures, and each step explained, as for example, **14**, in which the pupils may be led to say: "16 units in 48 units, 3 times; 16 units in 48 tens, 3 tens or 30 times; 16 units in 48 hundreds, 3 hundreds

or 300 times; 16 units in 48 thousands, 3 thousands or 3000 times." Some of the exercises should be performed by partition as in **1**: $\frac{1}{2}$ of 40, $\frac{1}{2}$ of 400, etc.

84 *Answers:* **1** 871. **2** 1414. **3** 1593. **4** 2347. **5** 1982. **6** 706. **7** 1299. **8** 3276. **9** 14521. **10** 17660. **11** 4581. **12** 6709. **13** $2509\frac{1}{7}$. **14** 3466. **15** 1226. **16** 13801. **17** 12562. **18** $316\frac{4}{7}$. **19** 8201. **20** 9102. **21** 1243. **22** 12439. **23** 5800. **24** 8098. **25** 276. **26** 568. **27** 342. **28** 172. **29** 112. **30** 258. **31** 763. **32** 54. **33** 63. **34** $40\frac{11}{48}$. **35** $514\frac{9}{24}$. **36** $1058\frac{9}{36}$. **37** $1123\frac{3}{42}$. **38** $821\frac{5}{31}$. **39** $474\frac{4}{29}$. **40** $4652\frac{4}{18}$. **41** $1574\frac{21}{32}$. **42** $790\frac{9}{57}$. **43** 59. **44** 88. **45** 61. **46** 25. **47** $201\frac{3}{4}$. **48** $115\frac{1}{5}$. **49** $84\frac{63}{80}$. **50** $3917\frac{11}{20}$. **51** $1540\frac{18}{30}$. **52** $1234\frac{45}{60}$. **53** $493\frac{31}{70}$. **54** $799\frac{36}{80}$. **55** $693\frac{34}{90}$. **56** 163. **57** $104\frac{3}{58}$. **58** $1138\frac{9}{24}$. **59** $923\frac{24}{34}$. **60** $717\frac{5}{29}$. **61** $1264\frac{12}{43}$. **62** $1055\frac{5}{38}$. **63** 27. **64** 2306. **65** 48. **66** $89\frac{30}{374}$. **67** $715\frac{28}{85}$. **68** $627\frac{1}{48}$.

85 *Answers:* **1** 10070. **2** $6410\frac{30}{57}$. **3** $168\frac{11}{47}$. **4** $3027\frac{18}{98}$. **5** $3280\frac{45}{98}$. **6** $5768\frac{10}{43}$. **7** $148\frac{81}{83}$. **8** $1227\frac{32}{64}$. **9** $19140\frac{5}{49}$. **10** 1030. **11** 2004. **12** 470. **13** 3005. **14** 1608. **15** 1020. **16** 26. **17** 368. **18** 275. **19** 26. **20** $1090\frac{10}{65}$. **21** 13030. **22** 38886. **23** 28684. **24** 164356. **25** 21834. **26** 14692. **27** 9068. **28** 6912. **29** 1719. **30** 2440. **31** 3280. **32** 10495. **33** 11076. **34** 57631. **35** 17526.

If the finding of fractional parts of numbers is not understood, teach the process with objects, using small numbers.

86 *Answers:* **1** 958. **2** 1480. **3** $12152\frac{9}{29}$. **4** $1093\frac{8}{82}$. **5** $327\frac{13}{18}$. **6** 3472. **7** 10200. **8** 68. **9** 125. **10** \$0.78. **11** \$13.50. **12** $\$49\frac{2}{38}$. **13** \$125.37. **14** \$45.75. **15** \$35.27. **16** 105 yds. **17** 1376 acres. **18** 837 pairs, \$0.70 left. **19** 897 quires.

Call attention to the fact that to have an entire number of parts in the quotient, the dividend and divisor must be of the same denomination; and that in getting the fractional part of a number

the answer is of the same denomination as the number wrought upon. Problems like **10** to **15** may be performed by taking the fractional part, thus:

$$\$74.10 = \text{cost of } 95 \text{ bu.}$$

$$\tfrac{1}{95} \text{ of } \$74.10 = \text{“ “ } 1 \text{ bu.} = \frac{\$74.10}{95} = \$0.78.$$

87 *Answers:* **6** \$3.25 \$40.625. **7** 20 kegs. **8** 70 T. **9** 6000 pkg. **10** \$221.25. **11** \$6.75. **12** 13 T. 14 lb. 10 oz. **13** 17 T. 9 lb. 11 oz.

Encourage originality in the solution of problems like **6**. Some pupils may find the cost of 1 cwt., and then of 5 cwt. Others may regard the 5 cwt. as $\frac{1}{4}$ of a ton and multiply by $6\frac{1}{4}$. Lead them to take the shorter way whenever it is clearly understood.

Lead the pupils to see that the same principle is involved in the addition of compound numbers as in the addition of simple numbers, the only difference being in the system of notation.

88 *Answers:* **1** 10 lb. **2** 7 lb. 1 oz. **3** 10 lb. 3 oz. **4** 8 T. **5** 10 T. 100 lb. **6** 15 T. **7** $1058\frac{13}{16}$. **8** 996 mi. **9** 293662. **10** \$4166.66$\frac{2}{3}$. **11** 357866. **12** 2286 ft. 634 ft. 365 ft. 11383 ft. combined height. **13** 3280 ft. **14** 177 ft. less.

89 *Answers:* **1** \$2724. **2** \$3364.28.. **3** \$95.19. **4** \$595.35. **5** \$46494. **6** \$11319. **7** 5401 gal. **8** 2894 ft. **9** 6 yr. **10** 152 mi. **11** \$1552.96. **12** \$234.

90 *Answers:* **1** \$586.258. **2** \$1750. **3** 84 casks. **4** $357\frac{8}{56}$ bars. **5** 245 yds. **6** 2297. **7** \$3841.25. **8** 48 tubs.

Some of the most difficult of the problems on the last pages of this section may need to be talked about in the recitation before they are given as a lesson for the pupils to learn. Such questions as the following for **2**, page 90, may be helpful in inducing the pupils to think, and in leading them to give a good written analysis: “What is given in this problem? What is required? Do you know the whole number of through passengers? How can you

find the number? How can you find the whole amount paid by these passengers?"

91–101 Whenever the conditions of a problem are not clearly understood, lead the pupils to grasp them by the use of questions and illustrations, and not by any set form of reasoning. Let the questions be such as to make the pupils think. In such problems as **7**, page 91, some preliminary questions like the following may be helpful: "10 pounds cost what part as much as 20 pounds? 4 pounds cost what part as much as 20 pounds? Do you see now any way of getting the cost of 8 pounds?" If the pupil is still uncertain, ask him what 4 pounds cost, and then what 8 pounds cost. In finding the cost of 25 pounds, the pupils may be led to find the cost of 1 pound first, and then of 25 pounds. There is some advantage in having pupils form a habit of working through the unit in finding the cost of a given number. But in such problems as **6**, page 91, it is better to work by multiples. To perform this problem the pupils should be led to see that 12 peaches will cost 6 times as much as 2 peaches.

In some problems it may be well, if the pupils find difficulty, to lead up to the required result by carefully-graded steps; for example, in **3**, page 92, to ask how many eggs would pay for 40 cents' worth of butter, 80 cents' worth, etc. In **1**, page 95, the questions might be: "How many can I make in 1 hour? in 3 hours?" And in **5**: "How many times can the measure be filled from a quart can? from a gallon can? from a two-gallon can?" In such problems as **4**, page 98, and **2**, page 99, it will be found useful to give the same conditions with small numbers. Formal oral "explanations" of problems should not be required at this time. Statements of processes, however, may be made, but care should be taken that the words exactly represent the thought of the speaker with little reference to the form of language.

Continued attention should be paid to the written analysis in the solution of problems. The following analyses of the last three problems on page 101 may suggest good forms for the pupils:

Given the number of bu. in 4 bins.
To find the number of lb. in 4 bins.

75	bu.		60 lb. in 1 bu.
48	"		248
90	"		480
35	"		240
248	"	in 4 bins.	120
			14880 lb. in 248 bu.

Given the height of an iceberg above water.
To find the whole height of the iceberg.

612 in. = height of iceberg above water.
8
4896 in. = " " " under "
612 in.
5508 in. = entire height of iceberg.

Given the cost of 1 ton of coal.
To find the cost of 87 tons.

$7.25 cost of 1 T.
87
5075
5800
$630.75 cost of 87 T.

SECTION VI.

NOTES FOR BOOK NUMBER FOUR.

A mastery of the subjects presented in Book No. 4 will give nearly all the practical knowledge of Arithmetic needed for the ordinary affairs of life, besides furnishing a good foundation for subsequent work. It may not be necessary for pupils to perform all the examples and problems here given; but before any considerable number of them are omitted, the teacher should be sure that

they are not needed, either for the purpose of fixing the principles and processes which have been taught, or for the purpose of mental discipline.

To more clearly understand the right use of the book, teachers are advised to read the Note to Teachers, in which are given its distinctive features and some hints as to possible dangers.

Appliances. — Some of the appliances needed for teaching the various subjects may be supplied by the teacher and pupils as they are needed, but it would be well to have at the outset all the common weights and measures, and plenty of cardboard or old pasteboard boxes from which discs and squares may be made.

Analysis of Problems. — No formal explanations or reasons should be insisted upon, but the method of solution should be frequently called for, both of oral and of written problems. The aim should be first, to have the pupils think as they solve the problem, and secondly, to have them express their thoughts in their own words. Good written forms of analysis should be required.

Development Work. — It has been the aim to give many simple exercises leading up to difficult processes or principles. If any problem is found too difficult for the pupils to perform, instead of attempting to "explain" the problem in words, teach the part not understood by the use of illustrations, or lead up to it by simple questions involving small numbers.

1–7 A few of these problems may be found too difficult to be performed orally, but let the pupils try them in that way before the pencil is taken. Not much development work ought to be needed for these review problems. Possibly in such problems as **7**, page 5, some questions like the following may be asked: "If there were 3 rows of trees and 4 trees in a row, and the trees were placed 20 ft. apart, how long and wide would the lot be in which the trees are planted? How many feet of fencing would be needed for such a lot? Suppose there were 8 rows and 6 trees in a row, how long and wide would the lot be? How many feet of fencing would be needed?" Such questioning may not be needed, especially if the pupils are required to draw an illustrative

diagram. **5**, page 6, is of a different kind, and may need such questions as: "How many miles in 1 hour? in 3 hours? in 8 hours? How many hours would it take him to walk 3 miles? 6 miles? 12 miles?" Simple, natural statements of processes should be expected, but see that they are not too wordy and labored.

8 *Answers:* **1** \$303.31. **2** \$643.38. **3** \$899.20. **4** \$1235.80. **5** \$1192.34. **6** \$6303.68. **7** \$3293.83. **8** \$5681.13. **9** 44214. **10** 174596. **11** \$2533.83. **12** \$2266.11. **13** \$1565.12. **14** \$1892.34.

9 *Answers:* **1** \$59.40. **2** \$317. **3** \$535.70 a mo. \$20.6038 a day. **4** \$657. **5** \$336. **6** \$57.80. **7** \$195.50. **8** \$5.18. **9** \$9.52. **10** \$0.24. **11** \$2. **12** \$60.75. **13** \$54.40. **14** 9.46 bbl. **15** \$115.39. **16** 2920 lb. $1\frac{3}{5}$ T.

10 *Answers:* **1** 25110 sq. ft. **2** 180 ft. **3** $1422\frac{2}{9}$ sq. yd. **4** \$697. **5** \$5.55. **6** \$66.24. **7** \$33.60 gain. **8** \$1561.91. **9** \$981.34. **10** \$1080.11. **11** \$1977.29.

11 *Answers:* **1** 8580. **2** 2630. **3** 10520. **4** 42900. **5** 44125. **6** 5150. **7** 30900. **8** 28504. **9** $3483\frac{1}{8}$. **10** 3914. **11** 1265. **12** 368. **13** 6 yr. 10 mo. **14** 243. **15** \$0.35. **16** 96 bu. **17** \$64.50. **18** 576 lb. **19** \$66.64. **20** \$7390.50. **21** \$4.44.

12 *Answers:* **1** \$123.70. **2** 17600 yd. **3** \$54.40. **4** 150 times 960 times. **5** 5000 min. **6** $55\frac{5}{9}$ yd.

In all problems on the last five pages of this section, require the pupils to write out statements of processes, to draw diagrams when needed, and to carefully label each result.

13 A brief exercise from the board to teach orders and periods may be given before the pupils answer the questions given on this page. The exercises on this page will suggest the kind of work to be given.

14 Much drill similar to this should be given from the board and by dictation. Observe the omission of the word "and" in

reading integers. Constant care in this regard will prevent confusion in the reading of mixed decimals.

15 *Answers:* **1** 4437920. **2** 44201673. **3** 3348952. **4** 11888983. **5** 7261543. **6** 5051010. **7** 1201991. **8** 990990991.

If the pupils have been thoroughly drilled in writing numbers, and if they know well the orders and periods, there will be little difficulty in performing these exercises.

16 *Answers:* **1** 79615294. **2** 2299711. **3** 5392649. **4** 71053996. **5** 1500635. **6** 174679994. **7** 17391047. **8** 44839931. **9** 357000879. **10** 56490661. **11** 271368. **12** 114480. **13** 183736. **14** 430008. **15** 2609088. **16** 5005625. **17** 2385184. **18** 255816. **19** 413100. **20** 359406. **21** 306000. **22** 154830. **23** 465885. **24** 674700. **25** 218772. **26** 350520.

If pupils thoroughly understand and can explain the four fundamental processes to millions, no explanation of those processes in the higher orders need be required.

17 *Answers:* **1** 9746388 18174424 21658640.
2 293900640 408789072 608285264.
3 9940860 11357820 22169520.
4 217914192 274410464 6505142176.
5 230799840 1141472235 3416445000.
6 574038864 558977328 1620167808.
7 585462987 553881996 797622465.
8 155670762 112108854 351392574.

9 184. **10** $127\frac{1}{32}$. **11** $174\frac{24}{45}$. **12** $130\frac{28}{35}$. **13** $93\frac{34}{78}$. **14** $1287\frac{42}{84}$. **15** $854\frac{81}{83}$. **16** $879\frac{10}{88}$. **17** $645\frac{1}{8}$. **18** $862\frac{8}{89}$. **19** $9445\frac{58}{72}$. **20** $6334\frac{53}{88}$. **21** $6835\frac{43}{78}$. **22** $4296\frac{6}{11}$. **23** $9633\frac{27}{89}$. **24** 200000. **25** 2000. **26** 200. **27** 3000. **28** 300. **29** 801. **30** 1045. **31** $1967\frac{140}{400}$. **32** $3010\frac{83}{90}$. **33** $851\frac{60}{600}$. **34** $4148\frac{40}{140}$. **35** $2115\frac{42}{180}$. **36** $1922\frac{90}{240}$. **37** $852\frac{345}{360}$. **38** $877\frac{30}{580}$. **39** $6392\frac{171}{604}$. **40** $18673\frac{284}{380}$. **41** $15014\frac{378}{843}$. **42** $10290\frac{70}{782}$.

43 $25742\frac{210}{378}$. **44** $7121\frac{137}{488}$. **45** $3561\frac{280}{308}$. **46** $333\frac{3313}{8307}$.
47 $442\frac{1484}{1800}$. **48** $431\frac{197}{1870}$. **49** $410\frac{558}{1428}$. **50** $92\frac{753}{643}$.
51 $483\frac{108}{436}$. **52** $830\frac{1340}{8372}$. **53** $870\frac{1084}{8800}$. **54** $1405\frac{1180}{5960}$.
55 $901\frac{8233}{8894}$. **56** $401\frac{3255}{8534}$. **57** $2423\frac{1185}{4088}$. **58** $762\frac{3589}{5037}$.

18 *Answers:* **1** 11514500 sq. mi. **2** 1940162 3571392 2994002. **3** 774 mi. 1278 mi. 525 mi. **4** 35.2 mi. Omaha, 12 h. 26 m. P.M. Tuesday; Cheyenne, 3 h. 38 m. A.M. Wednesday; Ogden, 8 h. 38 m. P.M. Wednesday; Palisade, 5 h. 17 m. A.M. Thursday; Reno, 1 h. 16 m. P.M. Thursday; Sacramento, 5 h. 38 m. P.M. Thursday; San Francisco, 8 h. 11 m. P.M. Thursday.

Other problems similar to these and to the problems on the following pages should be given, the object being to combine the getting of useful information with work in Arithmetic. The information may be gathered from the regular text-books, almanacs, newspapers, and railroad guides.

19 *Answers:* **1** \$162065. **2** \$121481. **4** \$1545777816. **5** \$2708393886 \$567934781 \$84886022 \$236244642 \$53642805 \$129756975 \$145141038 \$54989193 \$1162616070.

For areas referred to in **3**, see tables given on pages 20 and 21.

20 *Answers:*

1	Area,	162038		Population,		17401545.
2		268620				8857921.
3		610215				11150675.
4		753540				22362279.
5		1601103				3059408.
6		3395516				62831828.
7	107.3	33.3	18.2	29.6	1.9.	

Other problems may be given from this table, particularly those relating to the area and population of the pupils' State in comparison with those of other States.

21 *Answers:* **2** Portugal, 138.3 England, 540.9 France, 187.8 Germany, 236.7 China, 289.3 Italy, 261.8 Japan, 271.3 Spain, 88.7. **4** \$137238663 4919764. **5** 951 h. $43\frac{5}{73}$ d.

22 *Answers:* **1** 70200000. **2** $179\frac{89}{109}$. **3** 6122423 15251345 115050 11597412 12676044. **4** $639148\frac{3}{11}$. **5** \$3174 gain. **6** 51003321.

23 Before each set of examples and problems dealing with a new fraction, one or more short teaching exercises should be given in which the fraction should be taught both by itself and in relation to fractions already known. One of the best means for teaching fractions is the disks made of wood, cardboard, or pasteboard. Those for the teacher might be six or eight inches in diameter, and those for pupils two or three inches in diameter. If they are not provided by the school authorities, the pupils, doubtless, would be able to assist the teacher in cutting them from old boxes. Means for marking and cutting should also be provided. If the pupils to be taught are few in number, a table could be used by the teacher for placing the disks in teaching the various operations; in case a large number have to be taught at one time, the teacher could use a flat surface, inclined in such a way as to permit all the pupils to see the disks upon it. There might be made grooves upon the surface to permit the disks to remain in place. In teaching, sometimes the teacher places the disks for the pupils to observe, and sometimes the pupils place the disks they have by direction of the teacher. The small disks also can be used by the pupils in preparation of a lesson.

The preliminary teaching lesson for the exercises on this page is indicated by the following questions, which are asked by the teacher as he places the disks: "I will cut this circle into two equal parts, as you see. What is this part called (holding up one of the parts)? this part (holding up the other part)? How many halves in one? $\frac{1}{2}$ and $\frac{1}{2}$ equals what? $\frac{1}{2}$ taken out of one equals what? If I take the half two times, what is the result? How many times is the half contained in the whole one? Now, to review, $\frac{1}{2}+\frac{1}{2}$? 1 less $\frac{1}{2}$? 2 times $\frac{1}{2}$? 1 divided by $\frac{1}{2}$?" The pupils ought to be ready now for the exercises on this page. If they find difficulty with the problems in inches and half inches, similar exercises with measures cut from cardboard or paper might be given.

24–27 These for rapid oral work. Do not leave them until the pupils are able to perform them rapidly. Before **9**, page 24, is attempted, a short teaching exercise should be given. It may be necessary to perform some of the exercises at first with the aid of measures. Other exercises, like **9**, page 25, may have to be illustrated by sticks or marks; but when this is done they should be reviewed without aids of any kind.

28–29 The teaching exercise here may be as follows: "I will cut this circle, as you see, in 4 equal parts. One of these parts is called one fourth. What is this part called (holding up one of the parts)? What is this part called (holding up another part)? How many fourths in the whole circle? How many fourths here (pointing to two parts arranged in the form of a half-circle)? How many fourths here (pointing to three parts)? $\frac{1}{4}$ plus $\frac{1}{4}$ plus $\frac{1}{4}$ plus $\frac{1}{4}$ are what (putting the parts together in the form of a circle)? 1 less $\frac{1}{4}$ is what (taking away one of the parts from the circle)? 1 less $\frac{2}{4}$? 1 less $\frac{3}{4}$? How many times have I taken $\frac{1}{4}$ (putting two of the parts together)? 2 times $\frac{1}{4}$ is what? How many times have I taken $\frac{1}{4}$ here (putting three of the fourths together)? What is three times $\frac{1}{4}$? 4 times $\frac{1}{4}$? How many times must I take $\frac{1}{4}$ to make $\frac{2}{4}$? to make $\frac{3}{4}$? to make a whole one? $\frac{1}{4}$ is contained in $\frac{2}{4}$ how many times? in $\frac{3}{4}$ how many times? in a whole one how many times? What else may we call this part of the circle (putting two fourths together)? How many fourths is $\frac{1}{2}$? $\frac{1}{2}$ less $\frac{1}{4}$ is what? 2 times $\frac{1}{4}$ is what? How many times is $\frac{1}{4}$ contained in $\frac{1}{2}$? $\frac{1}{2}$ of $\frac{1}{2}$ is what?" Go on in this way comparing $\frac{1}{2}$ with $\frac{3}{4}$ and $1\frac{1}{4}$, also $\frac{3}{4}$ with $1\frac{1}{4}$ and $1\frac{1}{2}$.

In teaching the expression of the fraction, lead the pupils to see that the denominator expresses the number of parts into which the unit is divided, and the numerator the number of parts taken. Give the pupils much practice in this. Finally, they should be able to answer the questions: What does the denominator express? What does the numerator express? They should also be able to illustrate by objects or marks their answers.

30 Few or many of these exercises should be performed by the

aid of disks according to the pupils' understanding of them. **11**, **12**, and **13** can best be performed by the aid of sticks, marks, or dots. Show the pupils how this may be done. After these have been performed objectively the pupils might be led to give little explanations of the solution thus: "$\frac{1}{4}$ of 8 is 2; $\frac{3}{4}$ of 8 is 3 times 2, or 6; 6 is $\frac{1}{2}$ of 2 times 6; 2 times 6 is 12. $2\frac{1}{4}$ is $\frac{1}{4}$ of 4 times $2\frac{1}{4}$; 4 times 2 is 8, and 4 times $\frac{1}{4}$ is 1; 4 times $2\frac{1}{4}$ is 9." Explain by examples what is meant by "reducing to lowest terms" as required in **7**.

31–32 These exercises ought to be performed readily without the aid of objects or preliminary questioning, unless in some of the applied problems the measures are not familiar. The steps of the solution should be given. Thus, in **9**, page 31: "$\frac{1}{2}$ from $\frac{1}{4}$ I cannot take; so I take $\frac{1}{2}$ from $1\frac{1}{4}$, leaving $\frac{3}{4}$. I took 1 from 8, leaving 7; 1 from 7 leaves 6. *Answer*, $6\frac{3}{4}$." And in **6**, page 32: "The horse eats 3 times $\frac{1}{2}$ peck or $1\frac{1}{2}$ pecks in 1 day. It will take as many days for him to eat $4\frac{1}{2}$ pecks as $1\frac{1}{2}$ is contained times in $4\frac{1}{2}$, $1\frac{1}{2}$ is contained in $4\frac{1}{2}$, 3 times. *Answer*, 3 days. Since $4\frac{1}{2}$ bushels is 4 times as much as $4\frac{1}{2}$ pecks, it will take him 4 times 3 days, or 12 days." Probably some teaching will be necessary for this problem, but the pupils should be permitted to try to solve the problem before any help is given. If they solve it by reducing the pecks to quarts, accept the solution.

33–34 For teaching eighths and their relations to fourths and halves teachers are referred to the note in which a method for teaching fourths was shown (page 62 of Manual). Essentially the same plan should be pursued here. After the teacher has taught by objects all the important facts, short dictation exercises might be given for the pupils to solve with disks at their seats. Such exercises as the following will suggest the kind of work which may be given: $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$ equal what? $\frac{1}{8}+\frac{1}{8}$? (Always expect the answer to be given in the simplest form, and if they are not so given, ask questions till the desired answer is obtained.) $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$? $\frac{1}{4}+\frac{1}{8}$? $\frac{1}{4}+\frac{1}{8}+\frac{1}{8}$? $\frac{1}{2}+\frac{1}{8}$? $\frac{1}{2}+\frac{1}{8}+\frac{1}{8}$? $\frac{1}{4}+\frac{1}{2}+\frac{1}{8}$? $\frac{1}{4}+\frac{3}{8}$? $\frac{3}{8}+\frac{1}{2}$? $\frac{5}{8}+\frac{1}{4}$? $\frac{7}{8}+\frac{1}{4}$? $1\frac{3}{4}+\frac{1}{8}$? $1\frac{3}{8}+\frac{5}{8}$? From 1

take $\frac{5}{8}$; $1-\frac{3}{8}$? $\frac{1}{4}-\frac{1}{8}$? $\frac{3}{4}-\frac{1}{8}$? $\frac{1}{2}-\frac{1}{8}$? $\frac{1}{2}-\frac{3}{8}$? $\frac{3}{4}-\frac{1}{8}$? $\frac{3}{4}-\frac{3}{8}$? $1\frac{1}{4}-\frac{1}{8}$? $1\frac{1}{8}-\frac{3}{8}$? $1\frac{1}{4}-\frac{3}{8}$? Multiply $\frac{1}{8}$ by 5; $\frac{3}{8}\times 2$? $\frac{5}{8}\times 2$? $\frac{3}{8}\times 4$? $\frac{3}{8}\times 3$? $\frac{5}{8}\times 4$? $\frac{7}{8}\times 2$? $\frac{5}{8}\times 3$? $\frac{5}{8}\times 4$? $\frac{3}{8}\times 6$? $\frac{5}{8}\times 6$? $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{4}$ of $\frac{1}{2}$? $\frac{1}{2}$ of $\frac{3}{4}$? $\frac{1}{2}$ of $1\frac{1}{2}$? $\frac{1}{4}$ of $1\frac{1}{2}$? How many times is $\frac{1}{8}$ contained in $\frac{3}{8}$? $\frac{5}{8}\div\frac{1}{8}$? $\frac{1}{4}\div\frac{1}{8}$? $\frac{1}{2}\div\frac{1}{8}$? $\frac{3}{4}\div\frac{1}{8}$? $1\frac{1}{4}\div\frac{1}{8}$? $\frac{3}{4}\div\frac{3}{8}$?

The first 18 exercises on page 34 ought to be performed first with the aid and afterwards without the aid of the cut at the top of the page.

35–36 Most of these exercises should be performed without objects, and the steps of the solution should be required, as in **12**, page 35: "2 times $8=16$; $\frac{3}{8}$ of $8=3$; $16+3=19$. 8 multiplied by $2\frac{3}{8}=19$." And in **15** on the same page: "$2\frac{1}{2}=\frac{5}{2}$; $\frac{1}{2}$ of $\frac{1}{2}=\frac{1}{8}$; $\frac{1}{4}$ of $\frac{5}{2}=\frac{5}{8}$; $\frac{3}{4}$ of $\frac{5}{8}=\frac{15}{8}$ or $1\frac{7}{8}$."

37–38 These exercises are supposed to be performed orally; but it may be well for the sake of clearness and the acquirement of a good form of analysis for future use to have the pupils write out the separate steps of some of the most difficult, as, for example, in **14**, page 37:

Given the cost of $\frac{3}{8}$ bu. apples.

To find the cost of 2 bu.

\$0.60 = cost of $\frac{3}{8}$ bu.	\$1.60 = cost of 1 bu.
0.20 = " " $\frac{1}{8}$ bu.	3.20 = " " 2 bu.

9, page 38. $60\frac{1}{2}$ yd. = entire piece.

$16\frac{1}{4}$ yd. $\times 3 = 48\frac{3}{4}$ yd. = yards cut off.

$60\frac{1}{2}$ yd. $- 48\frac{3}{4}$ yd. $= 11\frac{3}{4}$ yd. remaining.

39 *Answers:* **1** $128\frac{1}{4}$ $65\frac{5}{8}$ $42\frac{3}{4}$ $58\frac{1}{8}$ $69\frac{7}{8}$. **2** $8\frac{1}{4}$ $43\frac{1}{8}$ $519\frac{3}{8}$ $105\frac{5}{8}$ $161\frac{1}{2}$. **3** $15\frac{15}{31}$ $189\frac{7}{8}$ $176\frac{1}{8}$ 868 $996\frac{7}{8}$ **4** $226\frac{7}{8}$ $62\frac{3}{8}$ $179\frac{3}{8}$ $229\frac{1}{2}$ $118\frac{1}{4}$. **5** $131\frac{3}{8}$ $55\frac{7}{8}$ 50 $259\frac{3}{8}$ $49\frac{3}{8}$. **6** 4800 $534\frac{3}{8}$ $163\frac{7}{8}$ 1050 $712\frac{1}{2}$. **7** $16\frac{5}{8}$ $96\frac{5}{8}$ $58\frac{5}{8}$ $353\frac{5}{8}$ $1353\frac{3}{8}$. **8** $60\frac{7}{8}$ $52\frac{1}{8}$ $251\frac{5}{8}$ 322 $46\frac{5}{8}$. **9** $2362\frac{1}{2}$ $49\frac{1}{8}$ $975\frac{3}{8}$ 720 33. **10** $152\frac{1}{8}$ $107\frac{3}{8}$ $95\frac{5}{8}$ $2766\frac{3}{4}$ 232. **11** $30\frac{3}{8}$ $66\frac{1}{8}$ 16 7875 $1714\frac{1}{2}$. **12** $80\frac{1}{4}$ $27\frac{1}{8}$ 380 $99\frac{1}{2}$ $72\frac{1}{2}$. **13** $456\frac{1}{8}$ lb. **14** \$$303\frac{5}{8}$. **15** 98 lots.

40 *Answers:* **1** 82 books. **2** 635¼ bu. **3** \$0.76. **4** 30 mi. **5** 6⅛ mi. **6** 22½ mi. 1676¼ mi. **7** \$267.75. **8** 4 times; 2¢. **9** Horse, \$645 Carriage, \$215. **10** \$1592½. **11** \$128¾. **12** \$7.56¼. **13** \$27.57⅜. **14** \$254.06¼. **15** \$100.78+. **16** \$562.18¾. **17** \$311¾. **18** 7.60½. **19** \$1.56¼. **20** \$1172½. **21** \$34.42½.

Good forms of written analysis are suggested by the following:

3

\$0.15½	selling price of 1 gal.
42	
30	
60	
21	
\$6.51	selling price of 42 gal.
5.75	cost " " "
\$0.76	gain.

9 \$860 = cost of horse and carriage = 4 times cost of carriage.
Cost of carriage = ¼ of \$860 = \$215.

41–42 One or more teaching exercises should precede this work. For suggestions as to how they may be conducted, see directions for teaching fourths and eighths. As soon as the pupils can discover for themselves that there are two ways of multiplying a fraction by a whole number and also two ways of dividing by a whole number, let them show objectively how this may be done, and afterwards state the facts and reasons. For example, the pupils should be led to show that ⅜ can be multiplied by 2 by increasing the number of parts while the size of the parts remains the same or by increasing the size of the parts while the number of parts remains the same. They may afterwards make the statement that a fraction may be multiplied by a number by multiplying the numerator or dividing the denominator by that number.

43 Exercises for practice, which should be performed at sight. If the pupils cannot perform them readily at first, let them work for a while with the disks. This will be good desk-work.

44 Care should be taken to make the work in division very simple at this stage of the pupils' progress. When the divisor is a fraction it should be contained in the dividend an even number of times. Lead the pupils to reduce the divisor and dividend to the same denomination before dividing. $1 \div \frac{1}{6}$, $\frac{3}{8} \div \frac{1}{8}$, etc., in **4**, should be solved by getting $\frac{1}{6}$, $\frac{1}{8}$, etc., of the given numbers.

When twelfths are taught, let the pupils have much practice in finding the relation of twelfths to sixths, thirds, halves, and fourths.

45 Show the pupils how this diagram can be used in the solution of the problems, and give similar ones. When they are understood let them be performed without aids of any kind.

46 The work in division here may be too difficult without some preliminary teaching. Besides the illustration given on page 45 for teaching division by a fraction, disks or squares may be used in which the divisor and dividend are made to be of the same denomination. Thus, $\frac{1}{2} \div \frac{1}{6}$, or $8\frac{2}{3} \div \frac{5}{6}$, may be represented objectively as the division of sixths by sixths. Until the pupils are entirely familiar with the process of division and of finding the fractional parts of numbers, lead them to illustrate with objects or drawings the operations called for.

47 Lead the pupils to state reasons in their own language first. Afterwards they can be led to improve their statements by some questions or suggestions from the teacher; or different pupils may be called upon to perform the same problem to see who will give the simplest and clearest explanation. By substituting whole numbers for fractions the conditions of a problem may sometimes be better understood; thus, in **3**, if the pupil hesitates or does not know whether to multiply or divide, say: "At \$2 a yard how many yards of cloth can I buy for \$8?"

49 In all cases where the problem can be solved at sight, do not permit the steps to be given. Answers to such exercises as, $\frac{1}{2} \div \frac{1}{4}$, $4 \div \frac{1}{8}$, and $\frac{1}{10} \times 2$, should be given at sight. In giving the steps of a solution some of the most obvious ones should be omitted, as, for example, in adding $8\frac{1}{2}$ and $4\frac{7}{10}$, it is enough for the pupil

to say: "$\frac{5}{10}$ and $\frac{7}{10}$ is $1\frac{1}{2}$, added to 12 is $13\frac{1}{2}$." And in multiplying $16\frac{3}{10}$ by 8 to say: "8 times 16 is 128, 8 times $\frac{3}{10}$ is $2\frac{2}{5}$, added to 128 is $130\frac{2}{5}$." Long and labored explanations should be avoided.

50 A table that can be used for drill in review at any time. Other exercises than those indicated can be given, as $E \times 6$, $C \times 12$, $I \div \frac{1}{2}$, etc. In denominate numbers the table can be used in many ways and to an almost unlimited extent. But in using drill tables care should be taken not to practice with them too much. On account of the ease with which drill can be given from them, the temptation is to use them for a longer time than is really needed for facility in the use of numbers.

51–52 Nearly all of these problems ought to be performed orally; but for the purpose of learning a good form of written analysis the pupils should be asked to write out in full the steps that are taken in the solution of the most difficult problems.

53 *Answers:* **1** $11\frac{1}{2}$. **2** $23\frac{1}{12}$. **3** $18\frac{1}{2}$. **4** $21\frac{3}{4}$. **5** $21\frac{5}{9}$. **6** $34\frac{1}{12}$. **7** $35\frac{2}{3}$. **8** 32. **9** $42\frac{1}{12}$. **10** $56\frac{4}{5}$. **11** $103\frac{1}{18}$. **12** $248\frac{4}{5}$. **13** $388\frac{2}{7}$. **14** $309\frac{3}{8}$. **15** $610\frac{8}{72}$. **16** $272\frac{3}{4}$. **17** $281\frac{7}{10}$. **18** $376\frac{7}{8}$. **19** $348\frac{3}{4}$. **20** $411\frac{1}{3}$. **21** $470\frac{5}{8}$. **22** $372\frac{3}{4}$. **23** $322\frac{1}{4}$. **24** $372\frac{5}{6}$. **25** $424\frac{1}{4}$.

It is not necessary in all cases to write out in full the solution of these problems, but only such parts of them as cannot be performed without the aid of figures. Thus, in **11** the pupil could say: "$\frac{8}{9} = \frac{2}{3}$, $\frac{4}{8} = \frac{1}{2}$, and $\frac{4}{12} = \frac{1}{3}$. Thirds, halves, and sixths can be reduced to twelfths. $\frac{2}{12} + \frac{3}{12} = \frac{10}{12}$, $+ \frac{4}{12} = \frac{14}{12}$, $+ \frac{6}{12} = \frac{20}{12}$, $+ \frac{8}{12} = \frac{28}{12} = 2\frac{1}{3}$. I write the $\frac{1}{3}$ and add the two, etc."

54 *Answers:* **1** $6\frac{1}{2}$. **2** $5\frac{1}{4}$. **3** $2\frac{2}{3}$. **4** $4\frac{1}{2}$. **5** $4\frac{1}{4}$. **6** $4\frac{5}{12}$. **7** $8\frac{1}{3}$. **8** $5\frac{1}{4}$. **9** $5\frac{1}{2}$. **10** $6\frac{3}{8}$. **11** $5\frac{3}{4}$. **12** $7\frac{1}{2}$. **13** $8\frac{5}{8}$. **14** $6\frac{3}{4}$. **15** $13\frac{1}{3}$. **16** $7\frac{3}{8}$. **17** $10\frac{7}{10}$. **18** $19\frac{2}{3}$. **19** $27\frac{1}{12}$. **20** $9\frac{7}{8}$. **21** $13\frac{5}{12}$. **22** $\frac{7}{8}$. **23** $10\frac{1}{2}$. **24** $45\frac{3}{8}$. **25** $1\frac{7}{10}$. **26** $33\frac{3}{4}$. **27** $9\frac{1}{2}$. **28** $62\frac{1}{3}$. **29** $12\frac{4}{11}$. **30** $55\frac{3}{7}$. **31** $8\frac{1}{10}$. **32** $50\frac{2}{3}$. **33** $24\frac{3}{8}$. **34** $21\frac{1}{3}$. **35** $44\frac{1}{3}$. **36** $51\frac{5}{8}$. **37** $28\frac{5}{9}$.

38 $66\frac{3}{7}$. **39** $159\frac{5}{8}$. **40** $86\frac{5}{8}$. **41** $141\frac{1}{12}$. **42** $223\frac{7}{9}$. **43** $533\frac{3}{10}$. **44** $607\frac{5}{8}$. **45** $220\frac{5}{8}$. **46** $188\frac{5}{8}$. **47** $286\frac{3}{7}$. **48** $377\frac{7}{12}$. **49** $319\frac{5}{8}$. **50** $389\frac{1}{12}$. **51** $163\frac{3}{8}$. **52** $353\frac{5}{8}$. **53** $652\frac{3}{10}$.

The same may be said of the solution of these problems as was said of the solution of the problems on the last page. Require the pupils to write out only such parts of the solution as cannot be performed orally; thus, in **35** the pupil may say, as he performs the problem: "$\frac{5}{8}$ from $\frac{1}{8}$ I cannot take; $\frac{5}{8}$ from $1\frac{1}{8}$ or $\frac{9}{8}$ = $\frac{7}{8}$; I write the $\frac{7}{8}$; 18 from 62, 44. *Answer*, $44\frac{7}{8}$." And in **41**: "$\frac{3}{8}$ from $\frac{3}{4}$. I can reduce them to twelfths. $\frac{9}{12}$ from $\frac{9}{12}$ = $\frac{1}{12}$, etc."

55 Reasons for steps taken should be given; thus, in **2**: "There are 4 fourths in 1; there are as many ones in 190 fourths as four fourths are contained times in 190 fourths or $47\frac{1}{2}$." The written form of solution at first may be: $\frac{4 \text{ fourths})190 \text{ fourths.}}{47\frac{1}{2}}$ This is a good time to look upon the fraction as only another form of division, the numerator being the dividend and the denominator being the divisor. With this view the only explanation needed is that one number is divided by another number. The reduction of mixed numbers to fractional numbers may be explained thus: "There are $\frac{3}{12}$ in 1; in 8 there are 8 times $\frac{3}{12}$ = $\frac{24}{12}$, + $\frac{5}{12}$ = $\frac{101}{12}$." If written, the form may be simply $8\frac{5}{12}$ = $\frac{24}{12}$, + $\frac{5}{12}$ = $\frac{101}{12}$.

In the solution of the exercises in multiplication on this page write out only such parts as cannot be carried easily in the mind; thus, in **33**: "3 times $2\frac{5}{6}$ = $8\frac{1}{2}$, $\frac{1}{2}$ of $2\frac{5}{6}$ = $1\frac{5}{12}$; $8\frac{1}{2}$ + $1\frac{5}{12}$ = $9\frac{11}{12}$. The written form which appears on slate or paper is

$$\begin{array}{r} 2\frac{5}{6} \\ \times 3\frac{1}{2} \\ \hline 8\frac{1}{2} \\ 1\frac{5}{12} \\ \hline 9\frac{11}{12} \end{array}$$ *Answer.*

56 *Answers:* **1** $1081\frac{1}{4}$. **2** $1956\frac{3}{8}$. **3** $1455\frac{1}{8}$. **4** $1615\frac{5}{8}$. **5** $1587\frac{1}{12}$. **6** $1398\frac{3}{8}$. **7** $2525\frac{5}{8}$. **8** $1916\frac{1}{4}$. **9** $1510\frac{3}{8}$. **10** $2877\frac{5}{12}$. **11** $2389\frac{3}{5}$. **12** $6197\frac{11}{12}$. **21** $11\frac{1}{8}$. **22** $13\frac{1}{2}$. **23** $10\frac{3}{5}$. **24** 12. **25** 18. **26** $21\frac{5}{8}$. **27** $41\frac{5}{12}$. **28** $80\frac{7}{12}$. **29** $192\frac{5}{12}$. **30** 228. **31** $346\frac{1}{2}$. **32** $339\frac{3}{5}$. **33** $177\frac{1}{7}$. **34** $219\frac{3}{4}$. **35** $145\frac{1}{8}$. **36** $248\frac{4}{11}$. **37** $79\frac{5}{11}$.

Pursue the same course with these exercises as was suggested for previous exercises. When necessary the divisor and dividend should be reduced to the same denomination. Instead of writing in full these denominations, the pupils may, after the first few exercises, write only the numerator; thus in **24**:

$$\begin{array}{r|l} 3\frac{1}{2} & 42 \\ \underline{2} & \underline{\ \ 2} \\ 7 & \underline{84} \\ & 12 \end{array} \quad \text{or } \tfrac{84}{2} \div \tfrac{7}{2} = \tfrac{84}{7} = 12 \textit{ Answer.}$$

$$12 \textit{ Answer.}$$

57 The answers to some of the exercises contain fractions having denominators larger than 12; but it is understood that the divisor and dividend when reduced to the same denomination may be treated as whole numbers in the division.

Answers: **1** $\frac{600}{12}$ $\frac{750}{12}$ $\frac{453}{12}$ $\frac{994}{12}$. **2** $4\frac{4}{7}$ $2\frac{8}{11}$ $3\frac{38}{101}$. **3** $13\frac{7}{9}$ $3\frac{5}{81}$ $1\frac{47}{77}$. **4** $10\frac{23}{30}$ $4\frac{73}{224}$ $3\frac{99}{290}$. **5** $921\frac{2}{3}$. **6** $14\frac{13}{20}$. **7** 146. **8** 405. **9** $880\frac{1}{2}$. **10** 12 steps. **11** 240 da. **12** $39\frac{2}{7}$ da. **13** 500 people. **14** 365 da. 313 working da. $\$61\frac{1}{12}$. **15** $6\frac{117}{120}$ min. **16** $\frac{1}{4}$ 2d boy earns 40¢ 3d boy, 30¢ 4th boy, 30¢. **17** \$1440 \$1080 \$1620.

Require the pupils to make drawings of the solution of problems whenever it is possible to do so, and to write out in full the steps of each illustrated solution. The principle involved in **16** is supposed to have been taught, but it may require some leading to get the pupils to illustrate the problem with dots as it should be. At first use simple denominations, as halves or thirds, gradually leading up to the work required. The finished drawing should represent the number of cents that each boy has earned. **17** can be performed in two ways: to first find the value of the whole lot and then $\frac{1}{3}$ of it; or, to multiply the value of $\frac{1}{12}$ of the lot by 4, first leading the pupils to see the ratio of $\frac{1}{3}$ to $\frac{1}{12}$.

58 *Answers:* **1** $1717\frac{1}{2}$ lb. \$4.29+. **3** $2411\frac{1}{4}$ mi. $3375\frac{3}{4}$ mi. **4** $7\frac{11}{12}$. **5** $\$4.37\frac{1}{2}$. **6** $13\frac{1}{2}$ h. $1\frac{1}{2}$ h. **7** 10 mi. $1\frac{3}{5}$ h. **8** 3 h. $18\frac{3}{4}$ mi. **9** 195 mi. $2\frac{5}{9}$ h.

Such problems as **9** can be illustrated by lines on the board drawn to scale, 1 inch to a mile, placing the distance above the line, and time below.

59 *Answers:* **2** 26 h. 15 min. **3** 12 in. entire $2\frac{3}{4}$ in. 1 in. **6** $\$5.16\frac{2}{3}$.

60–61 Stiff paper or cardboard should be used for these exercises. Some preliminary exercises may have to be given to lead the pupils to see clearly the relative size of the large square, the strips, and small squares, and to be able to express the size in decimals as required. Such exercises as the following may be useful: "How many strips in the large square? What part of the large square is each strip? 2 strips are what part of the large square? 3 strips are what part of the large square? Hold up 4 tenths of the large square. Hold up 7 tenths of the large square. Hold up 9 tenths of the large square. Each strip is how divided? Each small square is what part of a strip? How many of the small squares are there in the large square? What part of the large square is each small square? Cut off 1 hundredth of the large square. 1 more hundredth. How many hundredths have you cut off? Cut off as many small squares as will make 7 hundredths of the large square. Hold up 1 tenth and 2 hundredths of the large square. Hold up 2 tenths and 4 hundredths of the large square. Calling the large square one, what may we call 1 strip? 5 strips? 6 strips? 1 small square? 3 small squares? 7 small squares? Hold up 4 tenths and 6 hundredths. How many hundredths in 1 tenth? in 3 tenths? in 6 tenths? How many hundredths in 2 tenths and 3 hundredths? in 4 tenths and 7 hundredths? Write 1 tenth in the common form. What is the denominator? Another way of writing 1 tenth is to write the numerator 1, and place a dot called a decimal point before it, thus, .1. A fraction whose denominator is ten, or some power of ten, is called a decimal fraction. In this decimal fraction which we have written what only is expressed? How may we know that the figure one stands for 1 tenth? Read these decimals:

.7; .8; .3. How should we express 10 tenths? Hold up the part of 1 which these fractions express: .4; .7; .6. When we wish to express hundredths we write the numerator in the second place to the right of the decimal point, thus, .04. This expresses what fraction? .38 expresses how many tenths and how many hundredths? Read the fraction as hundredths."

After such an exercise the pupils should be ready to take up the work given on these pages. The last five exercises of page 61 should be extended until the reading and writing of decimals to hundredths are thoroughly understood.

62 Thousandths should be taught in a way similar to that suggested above for teaching hundredths. Great care should be taken in marking and cutting the squares and strips, each expression of figures representing what the pupils actually see to be the fractional part of the given unit.

63 Dwell upon each step here given until the pupils thoroughly understand it.

64 In **4** lead the pupils to see that $\frac{3}{4}$ or $\frac{1}{4}$ of 1 is the same as $\frac{3}{4}$ or $\frac{1}{4}$ of $\frac{100}{100}$ or $\frac{1000}{1000}$. $\frac{3}{4}$ of 100 hundredths is 75 hundredths; how expressed? etc. The reduction asked for in **5** should be performed as simply as possible; for example: "250 thousandths is how many hundredths? 25 hundredths is the same as what fraction?" If the pupils hesitate, ask what 50 hundredths is equal to, and then 25 hundredths. In **6**, **7**, and **8** lead the pupils to get the required fractional part of 100 hundredths or 1000 thousandths as before. Give other exercises similar to **10**, that the principle involved may be fully understood.

65 *Answers:* **10** 54087.645. **11** 376.848. **12** 656.647. **13** 123.728. **14** 407.503.

The first four exercises are important, and may be extended profitably. Addition of decimals ought not to be difficult to pupils who have been carefully trained previously. Let the explanations be as simply given as was advised for addition of whole numbers. The sum 24 thousandths to be treated as hundredths, and thou-

sandths ought to be as well understood as so many tens, to be treated as hundreds and tens.

66 *Answers:* **12** 5.858. **13** .063. **14** 15.731. **15** 27.251 74.066. **16** 640.991 76.324. **17** 65.147 78.591. **18** 85.201 86.308. **19** 290.317 26.699.

If necessary extend the objective work in subtraction of decimals. Simple explanations of steps in written work should be made by the pupils; thus, in **15**: "0 thousandths from 1 thousandth is 1 thousandth; 1 is equal to 10 tenths, and 1 tenth is equal to 10 hundredths; 5 hundredths from 10 hundredths is 5 hundredths; 7 tenths from 9 tenths is 2 tenths; 0 from 27 is 27. *Answer,* 27.251."

67 *Answers:* **1** 10.276 T. **2** .525. **3** .495 T. **4** 371.25 gr. **5** 27.4 gal. **6** 79.255 A.

If necessary, let the objects be used in multiplying.

68 *Answers:* **1** .32. **2** .032. **3** .216. **4** 1.184. **5** 2.422. **6** 1.9. **7** 6.75. **8** 552. **9** 1.218. **10** 3.084. **11** 5.715. **12** 5.201. **13** 5.272. **14** 6.68. **15** 5.022. **16** 9.048. **17** 13.344. **18** 10.89. **19** 18.252. **20** 20.925. **21** 25.05. **22** 39.088. **23** 73.104. **24** 102.942. **25** 151.552. **26** 255.948. **27** 268.584. **28** 405.405. **29** 691.01. **30** 5918.4. **31** 1168.50. **32** 2577.748. **33** 46840.8. **34** 2138.752. **35** 29238.48. **36** 1762.592. **37** 4350.675. **38** 1865.528. **39** 5659.316. **40** 3645.32.

69 *Answers:* **1** 5049.156. **2** 4750.395. **3** 2200.44. **4** 6796.998. **5** 7315.934.

Answers to nearly all the remaining problems should be written without any figures of solution. In all cases where the multiplier is 50 or 25, first multiply by 100 by removing the decimal point two places to the right, and divide by 2 or 4.

70–71 Dividing any number by an integer is simply getting a fractional part of the number, the product being of the same denomination as the multiplicand. The so-called process of

"multiplying by a fraction" is also getting the fractional part of a number. Thus, 8.4 × .9 may be read 9 tenths of 8.4. If this is not clearly understood let the pupils use objects as indicated.

72 In every operation with the objects let the process and result be represented by figures. Pupils will readily see in working with objects that when the divisor is a fraction both dividend and divisor are to be of the same denomination before dividing.

73 Problems in which large numbers are used, as in **10, 19, 20, 22,** need not be solved objectively. If the divisor is an integer it will be sufficient to get the fractional part of the number reduced to tenths, hundredths, or thousandths, and if the divisor is a fraction, to reduce both dividend and divisor to the same denomination; thus, in **10** the pupils may say 4 thousandths in 60000 thousandths 15000 times. And in **22**: $\frac{1}{120}$ of 120 tenths is 1 tenth. These solutions would be written as follows:

$$.004\overline{)60.000}\quad 15000. \qquad\qquad 120\overline{)12.0}\quad .1$$

74 *Answers:* **1** 6.1 3010 3010 16.5 403.5. **2** 2.46 780 340 200 .09. **3** 755 600.1 101.5 50 2.5. **4** .04 25.5 2 400 .9. **5** 3.2 .18 160. 17.8. .15. **6** 1541.5 120.48 .438 2166.4 9556. **7** 11760 1010 87.5 10.25 27.738. **8** 142.048 2360 27 3.2 3002. **9** 23.008 120.355 .979 2890 39.606. **10** 244.664 175 3.772 325 193.04. **11** 13.635 792 249.66 9.1224 1000.

The pupils see in multiplying that the multiplication of tenths by units gives tenths, that tenths of tenths are hundredths, and that tenths of hundredths are thousandths; and from these facts are able to make and follow a simple rule for pointing off. In the same way rules made for pointing off in division may be followed, but in all cases the pupils should be ready to give reasons for pointing off as they do. **12** and **19** should be performed by removing the decimal point to the left. Answers only of such problems need be written.

75 These exercises should be performed orally. To divide by 50 or 25 divide by 100 and multiply the quotient by 2 or 4.

76 *Answers:* **1** 110 yd. 1.1 yd. **2** 20 paces. **3** \$3 15¢. **4** 20¢ \$1.28. **5** 20. **6** 5. **7** 72. **8** \$15. **9** \$1300. **10** \$55.04. **11** 2000 pencils. **12** \$36000 \$44000 \$4800. **13** 4.08. **14** 792 66. **15** 792 ft. 107.25 ft. 303.60 ft. **16** 4 rd. 86 rd.

These problems should be performed without much figure work, unless the steps of the solution are required.

77 After distances are known by measurement, they should be placed upon the board and kept there for convenience of comparison. Other known distances, such as the distance from one town or city to another, might also be posted for reference.

78 *Answers:* **7** 2 mi. 1 mi. 2 mi. 4 mi. 3 mi. 40 rd. **8** 275 ft. 17160 ft. 31680 ft. 74.25 ft. **9** 83⅓ ft. 204.75 ft. 9240 ft. 235½ ft. **10** 413⅞ yd. 6600 yd. 20 yd. **11** $1\frac{88}{88}$ mi. $1\frac{13}{20}$ mi. 17½ mi. $14\frac{88}{88}$ mi. **12** 61½ ft. 20½ yd. \$2.56.

Most of these problems can be performed orally by the pupils. Steps and reasons for steps should be called for occasionally. After a problem is performed, and the correct answer given, questions might be asked; as for example in **12**: "What do you find first? How do you find it? How reduce to yards? Why? How do you find the cost? Why?" Generally, however, a full statement of the process should be given by the pupil without interruption.

79 In finding the square contents of a surface, lead the pupils to think first of the number of units in a row, and to multiply this number by the number of rows, as was shown in Book III.

80 If the pupils have not had previous practice in drawing to scale in connection with other studies, some time may well be spent upon it here. These exercises will suggest other work of the same kind which may be given. An explanation of **10** might be somewhat as follows: "There are 12 ft. in 144 in. and 8 ft. in

96 in. In a surface 12 ft. long and 1 ft. wide there are 12 sq. ft. In a surface 12 ft. long and 8 ft. wide there are 8 times 12 sq. ft., or 96 sq. ft."

81 In such problems as **1**, **5**, and **6**, lead the pupils first to reduce the dimensions to like terms; thus: "20 yd. = 60 ft. 60 sq. ft. × 20 = 1200 sq. ft." This problem would be explained as previously shown. In **7**, let the pupils find first the number of square yards that will exactly cover the seat, and afterwards the amount which would be required if in covering there were a waste of 6 in. in the length and width. In **9**, let as many pupils as can, make the proper allowance for corners in finding the area of the walk. Require plan to be drawn.

82 *Answers:* **1** 320 steps. **2** 62 trees. **3** 30375 sq. ft. 3375 sq. yd. **4** 5103 sq. ft. **5** 351 sq. yd. **6** 170 sq. ft. 57 ft. **7** $23894\frac{1}{6}$ sq. ft. $625\frac{3}{8}$ ft. **8** \$3361.16 \$32.77. **9** $5\frac{5}{6}$ rd. **10** $27\frac{1}{12}$ mi. $86\frac{1}{2}$ mi. $64\frac{3}{4}$ mi. **11** 25 sq. yd. 25 yd.

Besides drawing a plan for the solution of each of these problems, the pupils should be expected to write out a brief analysis. The following solution of the last part of **8** may serve as an example of what may be done:

$178\frac{1}{2}$ ft., length of lot.	\$0.20 cost of 1 yd.
2	$163\frac{5}{6}$
357 ft., length of two sides.	\$3260
$134\frac{1}{2}$ ft., width of lot.	$16\frac{2}{3}$
3) $491\frac{1}{2}$ ft., length of two sides and end.	$32.76\frac{2}{3}$ cost of fence.
$163\frac{5}{6}$ yd., length of fence.	

83 *Answers:* **1** $13\frac{1}{8}$ sq. ft. **2** $6\frac{1}{4}$ sq. ft. $12\frac{1}{4}$ sq. ft. **8** $10\frac{2}{15}$ in. **9** $41\frac{2}{3}$ ft. **10** 15 ft. **11** 90 ft. **12** 128.125 sq. ft. 432.25 sq. ft. 14.236$\frac{1}{3}$ sq. yd.

84 A teaching exercise upon angles and right angles should precede these exercises. If the teacher's definition of angle is the difference of direction of two lines, he places upon the board pairs of lines extending in different directions, and shows that the angles

are of different sizes. In all these examples the lines forming the angles should be represented as meeting, and the point where the lines of the angle meet is called the vertex. Proceeding, the teacher draws two lines crossing each other, thus, +, and says, when two lines cross each other so as to make four equal angles, the angles are called right angles. The pupils are led to describe an angle as the difference of direction of two lines, and a right angle as a square corner. Angles similar to those in **1** are drawn, and the pupils are led by questioning to say that an obtuse angle is an angle greater than a right angle, and that an acute angle is an angle less than a right angle.

Before parallelogram is taught, parallel lines are drawn, and described as lines having the same direction, or, as lines which are as far apart in one place as in another, or, as being the same distance apart. Then a figure is drawn whose opposite sides are parallel, and the figure is described as it was drawn, the description being drawn from the pupils by questioning. The terms base and altitude are also taught, and the pupils are led to point out the base and altitude of several parallelograms. Parallelograms with right angles and those that are not so formed should be drawn, and the pupils should be told that parallelograms with right angles or square corners are rectangles. They should be led to see that all figures whose square contents they have thus far found are rectangles. They are now ready for the work designated in **2** and **3**, which should be repeated until they see that the area of any parallelogram can be found by multiplying the length by the width, or the base by the altitude.

85 Applications of the principle developed in the last lesson should be made until the pupils can readily find the area of any parallelogram. The "diamond" used in playing baseball is a square whose edge is 90 ft.

As the pupils proceed to find the relative size of a parallelogram and triangle of the same base and altitude, do not assist them. Simply ask the question at first, and give directions for finding the answer only as they are found to be necessary. The statement

in answer to the question contained in **5** should be made by the pupils themselves.

86 *Answers:* **4** 26400 sq. ft. **5** 8 ft. 6 ft. 6 ft. 4 ft. 36 sq. ft. **6** 9 ft. **7** 180 sq. ft. 20 sq. yd. **8** 20 sq. yd. **9** 80 yd.

No assistance should be given the pupils beyond what is given in **1**. It would be well to give several problems in finding the area of triangles. It will be profitable and agreeable to the pupils to give them problems similar to **5**. Give measurements of such figures by dictation; say that the figures are drawn to a given scale, and ask for the area. The following dictation exercise may serve as a model: "Draw a line *AB* 1 inch long. From the point *A*, perpendicular to *AB*, draw a line 3 inches long. Mark the end of this line *C*. Join *BC*. From a point *D* in the line *AC*, 2 inches from *A*, draw a line parallel to *AB*. Mark the end of line last drawn *E*. Supposing these lines to be drawn to a scale of 12 rods to an inch, find the area of each figure drawn." Similar figures drawn on the board, representing fields, gardens, etc., will give interesting work to pupils.

87 *Answers:* **2** \$16.80. **3** 224 ft. 1536 sq. ft. 107⅔ sq. yd. **4** 6 sq. ft. **5**. 108 sq. ft. 162 sq. ft.

A teaching exercise may be necessary before the pupils can find the areas of *a*, *b*, and *h*. It may be sufficient to ask them to make light dotted lines so as to make each of the figures to consist of a rectangle and a right triangle. Work similar to **1** may be given both as drill and test exercises. Give as little direct assistance as possible to the pupils in finding these areas. In addition to plans the pupils should write out in full the solution of the last four exercises.

88 *Answers:* **1** 15 ft. **2** 3600 sq. ft. **3** 27⅛ sq. yd. **4** 60 sq. yd. \$14.40. **5** 128½ sq. ft. **6** 22 sq. ft. **7** 9¼ sq. ft. **8** 22500 sq. ft.

The pupils should draw a plan illustrating the shape and location of the lot upon the street, in the solution of **8**. Let the pupils

be free to choose the conditions of required original problems. Encourage them to give as difficult problems as they are able to solve.

89 Show to the pupils as many of these coins as can be obtained. Of the coins mentioned the gold three-dollar piece, the gold dollar, and the silver half-dime are not now coined. The silver three-cent piece also is not coined. The number of cents in fourths, eighths, fifths, thirds, and sixths of a dollar, should be remembered, and problems involving these sums should be performed orally.

90 This account should be copied, and each entry explained before the account of the following week is written. Balance on hand in **2**, \$2.55; in **3**, \$111.89.

91 Require the pupils to make the ruling for a bill; also to copy and finish the bill given on this page. Explain the form of dating and receipting bills. The amounts of bills are as follows:

1 \$17.71. **2** \$1338.70. **3** \$413.50. **4** 1294.50. **5** \$69.46. **6** \$1801.75.

92 *Answers:* **1** \$1444.80. **2** \$4615.92. **3** \$3465.60. **4** \$464.75 \$302.40. **5** \$21.87. **7** 25*s.* 72*s.* **8** 4 32 25. **9** £3 £4¼. **10** 138*s.* 175*s.* **11** 5 sov. **12** \$.243. **13** \$1.458 \$3.645 \$7.29 \$17.496 \$42.525 \$59.778 \$10.8135.

As many of the English coins as can be obtained should be brought into the class, and their value given. Show the greater convenience of our decimal system both in writing and in reckoning.

93 Most of these problems can be performed orally by the pupils. Reasons for multiplying or dividing in reduction may or may not be given.

94 *Answers:* **7** 10 lb. 5 oz. **8** 15 lb. 6 oz. **9** 26 lb. 11 oz. **10** 18 lb. 7 oz. **11** 7 cwt. 50 lb. **12** 11 cwt. **13** 4 cwt. 20 lb. **14** 8 cwt. 70 lb. **15** 4 T. 800 lb. **16** 10 T. **17** 9 T. 100 lb. **18** 17 T. 300 lb. **19** 2 lb. 10 oz. **20** 1 lb. 12 oz. **21** 1 lb. 9 oz. **22** 4 lb. 6 oz. **23** 1700 lb. 12 cwt. 600 lb. **24** 14 cwt.

1600 lb. **25** 1 T. 16 cwt. **26** 2 T. 14 cwt. **27** 4 T. 11 cwt. **28** 1 lb. **29** 1 lb. 8 oz. **30** 1 cwt. **31** 2 cwt. 40 lb. **32** 1 T. 4 cwt.

As many as possible of these problems should be performed orally, but for the sake of exercise in analysis the solution may be written. Explanations should be the same as in addition and subtraction of simple numbers.

95 *Answers:* **5** 25 lb. 8 oz. **6** 14 lb. **7** 13 lb. 11 oz. **8** 11 lb. 4 oz. **9** 66 lb. 12 oz. **10** 43 lb. 5 oz. **11** 118 lb. 2 oz. **12** 142 lb. 8 oz. **13** 13 T. **14** 33 T. 4 cwt. **15** 15 T. 1800 lb. **16** 33 T. 4 cwt. **17** 37 T. 10 cwt. 60 lb. **18** 37 T. 12 cwt. 64 lb. **19** 95 T. 11 cwt. 20 lb. **22** 4 lb. 8 oz. **23** 4 lb. 8 oz. **24** 2 lb. $10\frac{2}{3}$ oz. $1\frac{1}{3}$ oz. 3 lb. 12 oz. 2 T. 5 cwt. 3 cwt. 60 lb. **25** 2 T. 6 cwt. 2 T. $13\frac{1}{3}$ cwt. 1 T. 5 cwt. 10 T. 17 cwt. 50 lb. **30** $120. **31** $69.

Explanations may be as in simple numbers; thus, in **9**: "12 times 9 oz. = 108 oz. = 6 lb. 12 oz. I write the 12 oz. and add the 6 lb. to the next product. 12 times 5 lb. = 60 lb. + 6 lb. = 66 lb. *Answer*, 66 lb. 12 oz." And in **24**: "$\frac{1}{3}$ of 24 lb. = 2 lb., and 6 lb. remainder = 96 oz. $\frac{1}{3}$ of 96 oz. = $10\frac{2}{3}$ or $10\frac{2}{3}$ oz. *Answer*, 2 lb. $10\frac{2}{3}$ oz."

96 These are oral exercises, which can also be given for desk-work, to be brought into the class. Explanations of processes might be given in the recitation.

97 *Answers:* **1** 5 bu. 3 pk. **2** 6 bu. 0 pk. **3** 11 bu. 1 pk. **4** 8 bu. 2 pk. 1 qt. **5** 12 gal. **6** 13 gal. 3 qt. **7** 11 gal. 2 qt. 1 pt. **8** 3 gal. 3 qt. 1 pt. 1 gi. **9** 11 gal. 1 qt. 0 pt. 3 gi. **10** 6 bu. 2 pk. 6 qt. **11** 7 bu. 2 pk. 5 qt. **12** 1 bu. 1 pk. 3 qt. $1\frac{1}{2}$ pt. **13** 1 pk. 2 pk. 4 qt. 3 pk. 6 qt. **14** 2 bu. 2 pk. **15** 2 bu. 3 pk. **16** 2 bu. 2 pk. **17** 3 pk. 6 qt. **18** 4 gal. 3 qt. **19** 3 gal. 3 qt. **20** 5 gal. 1 qt. **21** 4 gal. 2 qt. 1 pt. **22** 1 gal. 1 qt. 3 gal. 1 qt. **23** 3 pk. 4 qt. 3 pk. 5 qt. **24** 1 pk. 1 pt. **25** 3 gal. 1 pt. $1\frac{1}{3}$ gi. 3 gal. 1 qt. $\frac{1}{3}$ pt.

26 8 gal. **27** 7 qt. **28** 7 pk. 4 qt. **29** 5 bu. 2 qt. **30** 25 gal. 2 qt.

These may be given for written desk-work, although the pupils ought to be able to perform them orally.

98 *Answers:* **1** 13 bu. **2** 34 gal. 2 qt. **3** 80 bu. 1 pk. **4** 21 gal. **5** 153 gal. 3 qt. **6** 99 bu. **7** 61 bu. **8** 152 gal. 2 qt. **9** 1 pk. 2 qt. $1\frac{1}{8}$ pt. **10** 2 qt. 1 pt. $1\frac{1}{8}$ gi. **11** 5 bu. 3 pk. **12** 7 gal. $\frac{3}{8}$ qt. **13** 8 gal. 1 qt. **14** 2 bu. 1 pk. **15** 7 pk. $4\frac{3}{8}$ qt. **16** 2 bu. $25\frac{5}{8}$ qt. **17** 2 gal. $1\frac{1}{2}$ qt. **18** 9 qt. 1 pt. $2\frac{1}{2}$ gi.

For variety of form the pupils might be led to say in such problems as **21**: "If I give to one person 3 pecks, I can give 3 bushels or 12 pecks to as many persons as there are 3 pecks in 12 pecks. There are four 3 pecks in 12 pecks. So I can give 12 pecks to 4 persons." But this or any one form should not be insisted upon.

99 *Answers:* **1** \$36. **2** \$15.50. **3** \$8. **4** \$1.92. **5** \$11.71$\frac{1}{2}$. **6** \$3.53$\frac{1}{2}$. **7** 5 bu. 3 pk. $6\frac{1}{2}\frac{1}{2}$ qt. **8** 90¢. **9** 160 sec. 405 sec. **10** 40 min. 45 min. 210 min. **11** 18 h. 9 h. 117 h. **14** 3 h. 30 min.

100–101 These problems, which may be recited orally in recitation, might be given for desk-work. In finding the difference of time in years, months, and days, let the time be given first in entire years or months; for example, in **1**, page 101: "From Feb. 22, 1732, to Feb. 22, 1799, it is 67 years. From Feb. 22 to Nov. 22 it is 9 months. From Nov. 22 to Dec. 14 it is 8 + 14 days or 22 days. *Answer*, 67 years, 9 months, 22 days."

102 *Answers:* **1** $7\frac{1}{2}$ r. 20 r. $26\frac{2}{3}$ q. 375 q. 10 r. **2** 48 r. 500 q. 120 sheets. $3086\frac{2}{3}$ sheets. **3** 98 r. 15 q. **4** 500 r. **5** 480 q. **6** 24 r. **7** \$17.50. **8** \$1795.32$\frac{2}{7}$. **9** 15 doz. $4\frac{3}{8}$ doz. **10** 44 doz. 105 doz. **11** 36¢. **12** 24 boxes. **13** \$90.

103 *Answers:* **10** \$3305.76. **11** \$10374.67. **12** \$2283.85.

If answers to exercises from **1** to **9** cannot be quite readily given, drill the pupils upon the exercises on page 1.

104 *Answers:* **1** 82. **2** 75. **3** 82. **4** 77. **5** 78. **6** 75. **7** 73. **8** 78. **9** 70. **10** 82. **11** 51. **12** 47. **13** 72. **14** 57. **15** 50. **16** 63. **17** 62. **18** 49. **19** 45. **20** 49. **21** 49. **22** 52. **23** 55. **24** 67. **25** 697. **26** 597. **27** 669. **28** 657. **29** 639. **30** 543. **31** 539. **32** 620. **33** 408. **34** 560. **35** 464. **36** 523. **37** 438. **38** 392. **39** 389. **40** 388. **41** 455. **42** 463. **43** 505.

Pupils should be required to add by lines as they are, and not write the numbers in columns before adding.

105 *Answers:* **1** 1800 bbl. **2** \$4412657.81+. **3** \$1272.90. **4** \$112. **5** \$346.18. **6** \$222.61.

106 *Answers:* **7** \$1.38. **8** 150 overcoats. **9** 73⅜ bu. **10** \$214.08. **11** 74 yd. \$68.45.

107 *Answers:* **9** \$14.02½ \$4488. **10** \$3.24 \$135. **11** \$10.33⅓ \$121.66⅔. **12** \$16.80 21 da. **13** 330 paces.

After working through the unit, as the pupils should be expected to at first, they might afterwards be led to get the desired result directly, as, for example, in **3**: "It will take 60 men ⅛ as long to do the work as it takes 20 men. ⅛ of 180 days is 60 days."

108 *Answers:* **1** 3 yr. 4 mo. 5 yr. 6 mo. **2** 2112 yr. **3** 8⅛ 83⅓ 330. **4** $7\frac{3}{8}\frac{3}{1}$ oz. **5** \$19.98. **6** 25 wk. **7** Friday. **8** 66 doz. **9** 24¢. **10** \$2240.

109 *Answers:* **3** \$299.25. **4** \$11.70. **5** C. Sea 42900 sq. mi. larger. **6** 3753 ft. higher. **7** \$1.50. **8** $76\frac{8}{3}\frac{8}{3}$. **9** \$360. **10** 7.2 mi. 8⅓ min.

110 *Answers:* **1** \$94.50. **2** 40 mi. 5 h. 45 min. **3** \$2.40. **4** \$84560. **7** 3576 sq. ft. **8** 1192 pickets. **9** 133⅓ sq. yd.

111 *Answers:* **1** 13½ sq. ft. **2** \$850. **3** \$3312. **4** 200 yd. **5** 456 ft. **6** \$573.60. **7** 18 min. 121½ min. **8** $1904213\frac{4}{1}\frac{4}{3}$ T. **9** 6748 poles. **10** 10000 cocoons $3409\frac{1}{11}$ mi.

SECTION VII.

NOTES FOR BOOK NUMBER FIVE.

Before taking up the work embraced in this book, the pupils are supposed to have a thorough knowledge of common fractions to twelfths and of decimal fractions to thousandths. They are also supposed to have had considerable practice in finding the areas of parallelograms and triangles, and in performing problems involving the common weights and measures. Teachers are advised to look over Book IV. to see if some parts of that book may not be reviewed before taking up the work of this book. Teachers are also referred to the Note to Teachers of Book V. for general suggestions.

1–2 There should be practice upon these exercises until answers are given with a good degree of promptness. At first it may be necessary to solve some of them by steps; for example, in **3**, page 2, the pupils may be led to say, in multiplying 47 by 8: "320, 56, 376." Or in **14**, page 1: "400, 85, 485; 300, 135, 435." But by degrees answers should be given at sight, or the steps should be so quickly taken as to make the exercises practically sight exercises. Previous practice should have made the pupils familiar with the products of all numbers to 20 by all numbers to 10, i.e., 136 should be recognized as the product of 17 and 8 as readily as 96 is recognized as the product of 12 and 8. The reverse operations in division should be equally familiar.

Some analysis may also be necessary in the solution of the exercises in fractions, but such analysis should be as limited as possible; for example, in **19**, page 2, the pupil may think and say "$\frac{2}{6}$, $\frac{4}{6}$, $\frac{2}{3}$, $7\frac{2}{3}$." After a while a less number of steps than are here given will be needed.

3–6 Short and naturally-expressed explanations of these exercises should be made by the pupils. The following will serve as

examples of what might be expected of them: **26**, page 3: "$62\frac{1}{2}¢ = \frac{5}{8}$ of a dollar. 10 bushels will cost 10 times $\frac{5}{8}$ of a dollar or $\$\frac{50}{8}$. As many dozen eggs will be worth this as $\$\frac{3}{8}$ is contained times in $\$\frac{50}{8}$, or $16\frac{2}{3}$ dozen." **18**, page 4: "3 rods $= 49\frac{1}{2}$ ft. A surface $49\frac{1}{2}$ ft. long and 1 ft. wide will contain $49\frac{1}{2}$ sq. ft. A surface of the same length 6 feet wide will contain 6 times $49\frac{1}{2}$ sq. ft. $= 240 + 54 + 3 = 297$ sq. ft." **16**, page 5: "30 apples will cost 10 times as much as 3 apples. 10 times 2 cents $= 20$ cents, cost. The boy would get 15 times as much for 30 apples as he gets for 2 apples. 15×3 cents $= 45$ cents. He gained 45 cents less 20 cents, or 25 cents."

7 *Answers:* **1** \$20415.16. **2** \$30446.15. **3** \$23953.38. **4** \$6807.89. **5** \$2685.25. **6** \$543.75. **7** \$278.64. **8** \$476.28. **9** \$980. **10** \$14.72. **11** \$216. **12** \$94.50. **13** \$22.68. **14** \$31.65$\frac{1}{2}$. **15** \$118.71. **16** \$294. **17** \$2033$\frac{1}{3}$, son; \$18300, widow. **18** \$2218$\frac{2}{3}$.

Let the pupils understand that in all final results of cost or selling price, when the fraction of a cent is less than half, the fraction is dropped, and when the fraction is one half or over one half, an extra cent is added.

8 *Answers:* **1** Total area, 51250800 sq. mi.; Total population, 1467920000. Population per sq. mi.: Europe, 101.3; Asia, 57.7; Africa, 11.03; Australasia and Pacific, 1.4; N. A. and W. I., 13.8; So. A., 5.3; Polar regions, .06. **5** \$11.25 \$15 \$10.35. **6** \$117 \$175.50 \$130. **7** \$57 \$49.59 \$42.75 **8** \$50.62$\frac{1}{2}$ \$70.87$\frac{1}{2}$ \$54. **9** \$224$\frac{1}{4}$ \$243$\frac{3}{4}$ \$260. **10** \$4009.50 \$3766.50 \$3098.25. **11** \$14700 \$21840 \$28560. **12** \$54 \$66 \$32. **13** \$180 pencils 900 pencils 2250 pencils.

Several problems may be made of **2**, **3**, and **4**. Let the pupils make as many as they can before the teacher suggests any.

9 *Answers:* **1** \$4000 profit. **3** 1600 oranges. **4** 127$\frac{3}{4}$ bu. \$76.65. **5** 89$\frac{1}{10}$ bbl. **6** \$93. **7** 14452.1. **8** 4435.33.

9 13960.22. **10** 111.607. **11** 5604.935. **12** 479.917. **13** 6066.925. **14** 28608 123586.56 288654.72. **15** 100 10 1. **16** 1600 240 32. **17** 1710 4788 4959. **18** 6.88 54544 35.088. **19** 5407.2 48.064 3652.864. **20** 7.68 3.84 .768.

10 *Answers:* **1** 36.72 3.672 4.08. **2** 2.38 357 23.8. **3** 39.67 3967 396.7. **4** 9.6 ft. **5** 10 panes. **6** \$3.00, cost of 1 bbl. \$9.75. **7** \$.20 \$2.95. **8** \$1522½. **9** 5 da. **10** 20 bbl. **11** 2000 pencils 166⅔ doz. **12** 3780 whites 294 colored 126 Chinese. **13** 174 sheep. **14** 495 doz. **15** \$21.21⅛. **16** 643½ bu. 21⅛ loads \$129.95⅛ profit. **17** 2 yr. 2 mo. 7 da. 1 yr. 9 mo. 15 da. **18** \$74.66⅔. **19** \$.95. **20** \$738. **21** 8.526 T.

11 Examples of the terms *odd number, even number, factor, prime factor, multiple,* and *least common multiple,* will have to be given before the exercises involving those terms are given. A good method is to write upon the board examples of what is desired to be taught, and then to ask the pupils to give other examples. After the pupils get a clear idea of the terms they may be asked to define them in their own words. The following definitions will suggest to teachers the kind of illustrations and questions that may be used. An even number is a number that is exactly divisible by 2. An odd number is a number that is not exactly divisible by 2. A factor of a number is its divisor. A prime factor is a divisor that is a prime number. A multiple of a number is any number which it will exactly divide. A common multiple of two or more numbers is any number which each of them will exactly divide. The least common multiple of two or more numbers is the least number which each of them will exactly divide.

12 The exercises of this page should be practiced upon until the pupils can give the required answers readily. The composite factors may be given first if necessary.

13 In teaching how to find the prime factors of large numbers, use small numbers first. Two ways of factoring may be used, — first to get the composite factors, and then the prime factors of

these; thus the prime factors of 240 may be found by finding the prime factors of 24 and 10, or 12 and 20. Another way is to divide the number successively by prime numbers thus:

2	240
2	120
2	60
2	30
3	15
	5

The prime factors of 240 are 2, 2, 2, 2, 3, and 5. Lead the pupils by examples to see that the common divisor of two or more numbers exactly divides each of the numbers; thus, in finding the greatest common divisor of 8 and 12 the question should be asked: "What number will exactly divide 8 and 12? Is there any other common divisor? Which is the greatest common divisor?" Other examples should be given of the same kind, the numbers increasing in size as the lesson proceeds. The kind and order of exercises here given will suggest to teachers what should be done before each new set of exercises is given. Let the work proceed slowly at this point, and review frequently.

14–15 The thorough knowledge of common fractions to twelfths, which pupils have acquired by objective teaching and much practice, will help them to use, intelligently, fractions of other denominations. By means of the cut at the head of the page, pupils may learn something of the relative size of fractions of various denominations. Lead them to express in fractional form any given part of the unit, indicating the fractional unit and the number of such units. By illustrations and comparisons such as are here shown, lead them to express a fraction by smaller terms. Lead them to see and to say that in such a change of expression, the size of the parts has increased while the number of parts has decreased. In the same way teach the expression of a fraction by larger terms, and why the value is not changed. Drill upon exercises similar to **17**, page 15, should be given until the pupils can tell at sight to what common denominator any two or more fractions, whose denominators are less than 20, may be reduced.

16–18 Frequently the common denominator of such fractions as are given in **1** and **2**, page 16, can be named at sight. If, however, the denominators are large, let their least common multiple

be found by finding all the different prime factors, as shown in the following illustrative solutions:

What is the least common multiple of 80, 72, and 60?

$80 = 2 \times 2 \times 2 \times 2 \times 5$
$72 = \cancel{2} \times \cancel{2} \times \cancel{2} \times 3 \times 3$
$60 = \cancel{2} \times \cancel{2} \times \cancel{3} \times \cancel{5}$
$80 \times 3 \times 3 = 720 = \text{L. C. M.}$

By this method the L. C. M. is the largest number multiplied by the factors of the other numbers not found in the largest number.

2	80	72	60
2	40	36	30
2	20	18	15
3	10	9	15
5	10	3	5
	2	3	1

By this method the numbers are divided by any prime number that will divide two of them without a remainder. The divisors, remaining quotients, and undivided numbers are the factors of the least common multiple.

$2 \times 2 \times 2 \times 3 \times 5 \times 2 \times 3 = 720.$

Constant drill upon the oral exercises given on these three pages should be given. A good method of drill is to let the pupils repeat the steps orally; thus, in **24**, page 16: "These fractions can be reduced to twelfths. $\frac{8}{12}$ and $\frac{3}{12}$ are $\frac{11}{12}$, and $\frac{10}{12}$ are $\frac{21}{12}$, and $\frac{6}{12}$ are $\frac{27}{12} = 2\frac{3}{12}$ or $2\frac{1}{4}$." By degrees the pupils may be led to state only results; thus in **47**, page 16: "$\frac{30}{36}$, $\frac{38}{36}$, $\frac{68}{36}$, $\frac{100}{36} = 2\frac{28}{36}$ or $2\frac{7}{9}$"; or even shorter than this: "30, 38, 68, $\frac{100}{36} = 2\frac{7}{9}$." Reduction to simpler forms and to mixed numbers may sometimes be made in the midst of the work; thus, in **7**, page 18: "$\frac{3}{4} + \frac{5}{6}$ less $\frac{5}{6} = \frac{3}{4}$." "$\frac{25}{30} + \frac{21}{30} = 1\frac{16}{30}$, less $\frac{10}{30} = 1\frac{6}{30}$ or $1\frac{1}{5}$." Short exercises of this sort in which every pupil is actively thinking should be given frequently.

19 *Answers:* **1** $2\frac{53}{60}$. **2** $4\frac{2}{9}$. **3** $2\frac{11}{28}$. **4** $2\frac{1}{2}$. **5** $5\frac{63}{80}$. **6** $2\frac{1}{120}$. **7** $1\frac{19}{21}$. **8** $1\frac{259}{264}$. **9** $\frac{51}{80}$. **10** $1\frac{34}{135}$. **11** $1\frac{69}{112}$. **12** $2\frac{51}{200}$. **13** $1\frac{113}{160}$. **14** $55\frac{203}{240}$. **15** $21\frac{35}{36}$. **16** $73\frac{1}{72}$. **17** $82\frac{101}{720}$. **18** $51\frac{211}{400}$. **19** $383\frac{79}{120}$. **20** $300\frac{7}{44}$. **21** $254\frac{167}{225}$. **22** $193\frac{203}{396}$. **23** $176\frac{629}{792}$. **24** $55\frac{53}{77}$. **25** $48\frac{44}{51}$. **26** $19\frac{149}{189}$. **27** $10\frac{49}{72}$. **28** $\frac{7}{20}$ $\frac{25}{32}$ $\frac{15}{68}$. **29** $\frac{23}{105}$ $\frac{63}{304}$ $\frac{13}{56}$. **30** $\frac{1}{24}$ $\frac{301}{576}$ $\frac{43}{210}$. **31** $\frac{34}{195}$ $\frac{67}{308}$ $\frac{217}{400}$. **32** $13\frac{24}{29}$ $24\frac{38}{45}$ $39\frac{31}{80}$. **33** $198\frac{26}{29}$ $109\frac{28}{85}$ $23\frac{17}{79}$. **34** $11\frac{18}{25}$ $14\frac{5}{11}$ $7\frac{28}{73}$. **35** $32\frac{13}{17}$ $164\frac{1}{30}$ $21\frac{38}{45}$. **36** $24\frac{1}{3}$ $45\frac{19}{22}$

$8\frac{13}{36}$. **37** $50\frac{6}{91}$ $99\frac{28}{35}$ $193\frac{7}{9}$. **38** $21\frac{7}{48}$ $66\frac{311}{380}$ $42\frac{113}{138}$. **39** $18\frac{17}{80}$ $11\frac{31}{96}$ 10. **40** $14\frac{13}{38}$ $17\frac{133}{336}$ $\frac{41}{66}$ $25\frac{35}{78}$. **41** $35\frac{39}{100}$ $6\frac{17}{24}$ $11\frac{1}{60}$ $41\frac{5}{48}$. **42** $33\frac{78}{90}$ $109\frac{43}{126}$ $91\frac{31}{42}$ $\frac{43}{49}$. **43** $11\frac{13}{28}$ $1\frac{39}{40}$ $46\frac{31}{80}$ $195\frac{41}{126}$.

Some of the above answers can be obtained without the aid of figures. Encourage the pupils to use as few figures as possible in the solution of problems.

The following forms of written solutions are suggested:

$$\begin{array}{r} 25 \\ 19\frac{5}{17} = \frac{15}{51} \\ 4\frac{28}{51} \\ 16\frac{2}{3} \\ \underline{8\frac{1}{8}} \\ 48\frac{44}{51} \end{array} \qquad \begin{array}{l} 27 \\ \frac{29}{144} + 1\frac{55}{72} + \frac{31}{36} + 7\frac{41}{48} = \\ \frac{29}{144} + \frac{110}{144} + \frac{124}{144} + \frac{123}{144} = \frac{386}{144} = 2\frac{98}{144} = 2\frac{49}{72} \\ 1 + 7 + 2\frac{49}{72} = 10\frac{49}{72}. \end{array}$$

20 *Answers:* **1** $30\frac{137}{154}$. **2** $85\frac{73}{120}$. **3** $8\frac{5}{24}$. **4** $5\frac{20}{21}$. **5** $108\frac{13}{87}$. **6** $\frac{13}{24}$. **7** $\frac{17}{27}$. **8** $25\frac{5}{7}$. **9** $52\frac{19}{21}$. **10** $3\frac{14}{45}$. **11** $15\frac{643}{1560}$. **12** $14\frac{49}{198}$. **13** $1\frac{9}{10}$. **14** $31\frac{57}{280}$. **15** $24\frac{1}{3}$. **16** $71\frac{13}{39}$. **17** $35\frac{1}{3}$. **18** $85\frac{1105}{1584}$. **19** $28\frac{7}{12}$. **20** $57\frac{11}{60}$. **21** $39\frac{55}{72}$. **22** $224\frac{121}{132}$. **23** $1\frac{27}{112}$. **24** $18\frac{59}{104}$. **25** $\frac{107}{315}$. **26** $26\frac{11}{105}$. **27** $\frac{23}{48}$ $1\frac{5}{126}$ $\frac{41}{60}$ $\frac{254}{273}$ $\frac{67}{120}$ $\frac{83}{140}$ $\frac{305}{378}$ $\frac{9869}{1573}$. **28** $\frac{61}{72}$ $1\frac{1}{6}$ $\frac{23}{24}$ $1\frac{47}{819}$ $\frac{29}{40}$ $1\frac{1}{55}$ $1\frac{25}{126}$ $1\frac{559}{968}$. **29** $4\frac{9}{32}$ $18\frac{23}{45}$ $6\frac{19}{45}$ $31\frac{379}{1547}$ $8\frac{61}{360}$ $4\frac{31}{130}$ $6\frac{13}{42}$ $8\frac{72}{91}$. **30** $8\frac{83}{216}$ $21\frac{29}{84}$ $18\frac{34}{55}$ $19\frac{811}{1547}$ $11\frac{47}{120}$ $18\frac{29}{120}$ $26\frac{725}{1008}$ $19\frac{2359}{3872}$. **31** $28\frac{67}{144}$ $30\frac{47}{99}$ $28\frac{79}{80}$ $52\frac{505}{1274}$ $46\frac{169}{840}$ $32\frac{9}{20}$ $31\frac{5}{84}$ $21\frac{1829}{2057}$. **32** $69\frac{15}{56}$ $89\frac{26}{48}$ $48\frac{24}{55}$ $68\frac{786}{2093}$ $58\frac{9}{40}$ $48\frac{709}{1120}$ $58\frac{101}{119}$ $80\frac{2305}{2783}$. **33** $1\frac{5}{144}$ $1\frac{3}{7}$ $\frac{317}{360}$ $1\frac{147}{163}$ $1\frac{1}{6}$ $1\frac{71}{154}$ $1\frac{37}{84}$ $1\frac{4553}{11144}$. **34** $4\frac{15}{32}$ $18\frac{4887}{6330}$ $6\frac{31}{90}$ $4\frac{3055}{3577}$ $8\frac{11}{18}$ $4\frac{62}{91}$ $6\frac{43}{74}$ $8\frac{72}{121}$. **35** $8\frac{247}{432}$ $21\frac{151}{189}$ $18\frac{5683}{6330}$ $20\frac{786}{3577}$ $11\frac{5}{6}$ $18\frac{1158}{1688}$ $27\frac{889}{4332}$ $19\frac{4321}{45576}$. **36** $28\frac{47}{72}$ $30\frac{1021}{1386}$ $28\frac{131}{144}$ $53\frac{225}{294}$ $46\frac{9}{14}$ $32\frac{26}{28}$ $31\frac{559}{1078}$ $22\frac{5445}{24431}$. **37** $69\frac{51}{112}$ $89\frac{263}{336}$ $48\frac{711}{1988}$ $69\frac{431}{483}$ $58\frac{2}{3}$ $49\frac{117}{224}$ $59\frac{154}{455}$ $81\frac{5537}{32289}$. **38** $4\frac{205}{288}$ $18\frac{9}{10}$ $6\frac{61}{72}$ $4\frac{7551}{10771}$ $8\frac{7}{9}$ $5\frac{671}{972}$ $7\frac{1}{18}$ $9\frac{8}{9}$. **39** $8\frac{222}{27}$ $21\frac{25}{27}$ $19\frac{111}{280}$ $20\frac{52}{153}$ 12 $19\frac{229}{264}$ $27\frac{67}{144}$ $20\frac{137}{352}$. **40** $28\frac{43}{48}$ $30\frac{18}{23}$ $29\frac{33}{80}$ $53\frac{187}{882}$ $46\frac{17}{21}$ $33\frac{7}{23}$ $31\frac{23}{36}$ $22\frac{91}{136}$. **41** $19\frac{44}{63}$ $89\frac{131}{144}$ $48\frac{372}{440}$ $69\frac{277}{1449}$ $58\frac{5}{8}$ $49\frac{1235}{2464}$ $59\frac{59}{102}$ $81\frac{1231}{2394}$. **42** $86\frac{31}{45}$ $94\frac{262}{315}$ $83\frac{103}{720}$ $79\frac{142}{315}$ $63\frac{11}{12}$ $70\frac{169}{220}$ $87\frac{3335}{4356}$ $99\frac{4}{11}$. **43** $12\frac{215}{864}$ $39\frac{73}{270}$ $24\frac{271}{315}$ $23\frac{20}{119}$

$19\frac{1}{3}$ $22\frac{103}{312}$ $32\frac{83}{144}$ $27\frac{325}{352}$. **44** $32\frac{995}{2688}$ $48\frac{103}{4608}$ $34\frac{831}{720}$ $56\frac{67}{1848}$ $54\frac{18}{63}$ $36\frac{7}{13}$ $36\frac{11}{12}$ $30\frac{38}{187}$. **45** $73\frac{889}{672}$ $107\frac{61}{240}$ $54\frac{181}{495}$ $72\frac{53}{2737}$ $66\frac{5}{8}$ $52\frac{2101}{2912}$ $64\frac{12}{17}$ $89\frac{36}{325}$. **46** $36\frac{187}{432}$ $51\frac{132}{594}$ $47\frac{143}{336}$ $71\frac{1131}{1848}$ $57\frac{10}{21}$ $50\frac{13}{24}$ $57\frac{47}{144}$ $41\frac{301}{5984}$. **47** $77\frac{89}{378}$ $110\frac{121}{432}$ $66\frac{202}{231}$ $87\frac{801}{2737}$ $69\frac{1}{2}$ $66\frac{487}{672}$ $85\frac{283}{2448}$ $100\frac{1267}{8056}$. **48** $97\frac{319}{1008}$ $119\frac{116}{528}$ $76\frac{2351}{2680}$ $120\frac{1125}{2254}$ $104\frac{13}{42}$ $80\frac{202}{224}$ $89\frac{31}{88}$ $102\frac{1877}{4301}$. **49** $114\frac{221}{720}$ $124\frac{323}{2310}$ $111\frac{31}{180}$ $130\frac{287}{480}$ $109\frac{11}{28}$ $102\frac{1}{8}$ $117\frac{455}{728}$ $120\frac{287}{1496}$. **50** $155\frac{33}{210}$ $183\frac{937}{5040}$ $130\frac{4818}{1920}$ $146\frac{619}{805}$ $121\frac{5}{12}$ $118\frac{429}{1120}$ $145\frac{89}{2528}$ $179\frac{265}{2024}$. **51** $\frac{3}{16}$ $\frac{11}{42}$ no ans. $\frac{188}{273}$ $\frac{53}{120}$ $\frac{31}{70}$ $\frac{99}{187}$ $\frac{627}{1573}$. **52** $\frac{35}{144}$ $\frac{8}{63}$ $\frac{181}{380}$ $\frac{8}{63}$ $\frac{1}{8}$ $\frac{131}{308}$ $\frac{7}{27}$ $\frac{5599}{12584}$. **53** $\frac{31}{72}$ $\frac{7}{8}$ $\frac{14}{40}$ $\frac{4676}{3733}$ $\frac{73}{120}$ $\frac{181}{220}$ $\frac{141}{189}$ $\frac{8305}{10648}$. **54** $3\frac{85}{95}$ $17\frac{287}{630}$ $5\frac{28}{30}$ $2\frac{341}{357}$ $7\frac{11}{18}$ $3\frac{235}{384}$ $5\frac{19}{27}$ $7\frac{149}{143}$. **55** $3\frac{125}{288}$ $17\frac{31}{90}$ $5\frac{167}{360}$ $2\frac{887}{1071}$ $7\frac{4}{9}$ $3\frac{63}{286}$ $5\frac{1}{9}$ 7. **56** $7\frac{209}{218}$ $20\frac{41}{54}$ $18\frac{48}{105}$ $19\frac{3059}{10829}$ $11\frac{11}{80}$ $18\frac{11}{120}$ $26\frac{269}{1008}$ $19\frac{1271}{3872}$. **57** $7\frac{29}{84}$ $20\frac{5}{8}$ $18\frac{33}{2520}$ $18\frac{500}{1071}$ $10\frac{3}{8}$ $17\frac{59}{264}$ $25\frac{675}{1296}$ $19\frac{128}{352}$. **58** $4\frac{89}{864}$ $3\frac{7}{270}$ $12\frac{518}{945}$ $15\frac{1282}{2023}$ $3\frac{2754}{12393}$ $14\frac{7}{2184}$ $20\frac{1}{48}$. **59** $27\frac{31}{56}$ $29\frac{67}{154}$ $28\frac{283}{720}$ $51\frac{137}{294}$ $45\frac{9}{14}$ $31\frac{102}{119}$ $30\frac{13}{108}$. **60** $27\frac{89}{144}$ $29\frac{61}{198}$ $28\frac{7}{240}$ $51\frac{2093}{6174}$ $45\frac{10}{21}$ $31\frac{18}{44}$ $29\frac{31}{36}$ $20\frac{1239}{1456}$. **61** $24\frac{318}{1003}$ $11\frac{53}{55}$ $22\frac{407}{720}$ $48\frac{853}{1666}$ $38\frac{1134}{3577}$ $28\frac{11}{33}$ $24\frac{3}{4}$ $13\frac{95}{363}$. **62** $20\frac{35}{432}$ $81\frac{671}{1782}$ $10\frac{9}{560}$ $32\frac{1017}{11662}$ $34\frac{17}{21}$ $14\frac{85}{408}$ $4\frac{49}{144}$ $21\frac{8425}{6656}$. **63** $64\frac{231}{234}$ $71\frac{7}{720}$ $42\frac{7}{495}$ $64\frac{1941}{2737}$ $50\frac{1}{18}$ $44\frac{1148}{2079}$ $52\frac{55}{1053}$. **64** $60\frac{187}{189}$ $67\frac{425}{432}$ $29\frac{179}{385}$ $48\frac{1831}{19159}$ $46\frac{185}{252}$ $30\frac{263}{672}$ $32\frac{317}{2448}$ $61\frac{1825}{3696}$. **65** $77\frac{118}{135}$ $72\frac{2588}{2835}$ $63\frac{3763}{10410}$ $59\frac{462}{1615}$ $51\frac{11}{12}$ $51\frac{79}{120}$ $60\frac{5233}{17424}$. **66** $16\frac{104}{105}$ $4\frac{1548}{1880}$ $34\frac{2231}{7920}$ $10\frac{209}{805}$ $5\frac{1}{19}$ $21\frac{299}{1120}$ $28\frac{4217}{24684}$ $17\frac{1681}{2376}$.

21 *Answers:* **1** $265\frac{5}{24}$ A. **2** $3154\frac{19}{21}$ bu. **3** $583\frac{5}{44}$ A. **4** \$.75. **5** $\$4169\frac{11}{24}$. **6** $\$626\frac{28}{80}$. **7** \$143. **8** $\$26\frac{18}{36}$. **9** $43\frac{55}{63}$ T. **10** $\$4\frac{38}{40}$. **11** $21\frac{9}{16}$ ft. **12** $557\frac{16}{21}$ ft. **13** $\$\frac{89}{120}$. **14** $\frac{1}{24}$.

Let the pupils perform orally such problems as they can perform in that way.

22 *Answers:* **1** $\frac{41}{56}$. **2** $194\frac{17}{36}$ lb. **3** $\frac{13}{84}$. **4** $39\frac{5}{8}$ bu. **5** $8\frac{7}{32}$ mi. **6** 5 h. 15 min. **7** $32\frac{23}{28}°$. **8** $30\frac{1}{8}$ yd. **9** $2\frac{1}{56}$ yd. **10** $\$8\frac{1}{40}$. **11** $618\frac{17}{60}$ mi. **12** $\$15\frac{5}{8}$. **13** $577\frac{1}{12}$. **14** $6986\frac{79}{325}$ gr.

Analyses both oral and written should be required, the steps to be clearly indicated, and what each result stands for. The method of solution in some cases might be indicated in one place and the work in another, as, for example, **4**:

100 bu. $-$ ($16\frac{5}{8}$ bu. $+$ $25\frac{3}{4}$ bu. $+$ $49\frac{1}{1\,8}$ bu. $-$ 127 pk.) = amt. of corn to fill the bin.

$16\frac{5}{8}$ 20
$25\frac{3}{4}$ 27
$49\frac{1}{1\,8}$ 22
90 $+$ ($\frac{8}{3\,6}$ = $1\frac{3}{3\,6}$) = $91\frac{1}{1\,2}$
$31\frac{3}{4}$
$60\frac{1}{8}$ bu.

$100 - 60\frac{1}{8} = 39\frac{7}{8}$ bu.

23 Let the pupils practice upon this illustrative work until they can clearly see and state the two ways of multiplying a fraction by a whole number.

24 Some of the work here called for is review, but it is advised that the objective work be continued until the pupils have a clear idea of the method of finding the fractional part of any number. After the lines have been used in the solution of problems, the same solutions should be reviewed without objects in such statements as the following (**14**): "$\frac{1}{5}$ of $\frac{3}{2}$ = $\frac{3}{1\,0}$ or $\frac{1}{8}$; $\frac{5}{8}$ of $\frac{3}{2}$ = 5 times $\frac{1}{8}$ = $\frac{5}{8}$." This could be followed by rapid silent solutions, the answers only being given.

25 Dividing by a whole number is only another form of expression for getting the fractional part of a number. $\frac{1}{2}$ of $\frac{4}{5}$ or $\frac{4}{5} \div 2$ is obtained in two ways, as shown. Let the pupils practice upon this with illustrations until they can tell readily which process to use.

26–27 Division by a fraction may also be taught by the aid of disks or by the drawing of lines or squares. For suggestive illustrative teaching see page 66 of the Manual. The same kind of illustrations may be used for division in other fractional numbers to show, first, that the number divided and the divisor are first subdivided into parts of the same size, or reduced to the same denomination. For a time this method of division may be employed. Afterwards the pupils, knowing that the quotient depends upon

the size of the divisor, may analyze problems as follows (**19**, page 27): "$\frac{5}{8} \div 1 = \frac{5}{8}$; $\frac{5}{8} \div \frac{1}{8}$ of $1 = 5$ times $\frac{5}{8}$ or $\frac{5}{8}$; $\frac{5}{8} \div \frac{3}{8} = \frac{1}{3}$ of $\frac{5}{8}$ or $\frac{5}{24} = 1\frac{1}{24}$." Such analysis should not be given to the pupils, but made by them in answer to questions from the teacher. After a time the pupils will see for themselves that the quotient obtained by dividing by any fraction is the same as the product obtained by multiplying by the same fraction inverted. There can be no harm in such a method so long as the pupils understand the process. In a final review of these pages let the pupils say, in such exercises as **12**, page 27: "$\frac{1}{8}$ of $4\frac{4}{7} = \frac{1}{8}$ of $\frac{32}{7} = \frac{4}{7}$," etc. And in such exercises as **15**, page 27: "$7 \div \frac{5}{3} = 7 \times \frac{3}{5} = \frac{21}{5} = 4\frac{1}{5}$." The processes of the solution may be made silently and only the answers given, thus: "$5 \div \frac{1}{8} = 15$; $\frac{1}{2} \div \frac{1}{4} = 2$."

28–30 Let the analysis of these problems be simply expressed, and, as nearly as clearness will permit, in the pupils' own words. The analysis of some of the problems may have to be preceded by questions; thus, in **2**, page 29: "*One* third of a yard will cost what part as much as *two* thirds? If you know the price of one third of a yard, how can you find the price of a yard?" And in **15**, page 29: "5 lb. will cost how many times as much as $2\frac{1}{2}$ lb.? $1\frac{1}{4}$ lb. will cost what part as much as $2\frac{1}{2}$ lb.?"

31 *Answers:* **1** $8\frac{4}{5}$ $80\frac{1}{2}$ 1062 $391\frac{2}{3}$ $656\frac{2}{3}$. **2** $81\frac{3}{4}$ $335\frac{3}{7}$ $272\frac{1}{2}$ 1845 $753\frac{3}{5}$. **3** $11\frac{7}{10}$ $20\frac{1}{4}$ $71\frac{1}{2}$ 100 $12\frac{3}{20}$. **4** $7\frac{1}{7}$ 8 $23\frac{1}{3}$ 45 $5\frac{7}{10}$. **5** $\frac{21}{100}$ $\frac{217}{1800}$ $\frac{74}{285}$ $\frac{13}{30}$ $\frac{117}{300}$. **6** $\frac{117}{180}$ $\frac{6}{35}$ $\frac{1}{8}$ $\frac{527}{540}$ $\frac{5}{9}$. **7** $24\frac{28}{33}$ $67\frac{41}{135}$ $179\frac{1}{5}$ $162\frac{123}{286}$ $164\frac{44}{45}$. **8** $232\frac{11}{27}$ $261\frac{2}{3}$ $292\frac{3}{7}$ $410\frac{11}{18}$ $1234\frac{7}{48}$. **9** $28\frac{11}{18}$ $455\frac{5}{8}$ $1943\frac{1}{4}$ $705\frac{3}{8}$ $426\frac{3}{5}$. **10** $134\frac{1}{18}$ $2542\frac{5}{8}$ $1945\frac{2}{7}$ $2604\frac{7}{8}$ $3539\frac{7}{12}$. **11** $149\frac{1}{3}$ $398\frac{1}{4}$ $410\frac{5}{8}$ $120\frac{3}{4}$ $33\frac{33}{48}$. **12** $847\frac{33}{52}$ $420\frac{21}{32}$ $7350\frac{33}{38}$ $2094\frac{7}{15}$ $773\frac{1}{28}$. **13** $187\frac{1}{4}$ $733\frac{3}{5}$ $888\frac{8}{9}$ $242\frac{2}{3}$ $96\frac{4}{5}$. **14** $281\frac{3}{16}$ $273\frac{11}{21}$ 2356 $18\frac{14}{57}$ $85\frac{1}{3}$. **15** 152 $253\frac{5}{8}$ $151\frac{3}{7}$ $409\frac{13}{16}$ $458\frac{11}{25}$. **16** $128\frac{1}{4}$ $2419\frac{7}{8}$ $1775\frac{9}{17}$ $1354\frac{5}{8}$ $1093\frac{1}{3}$. **17** $130\frac{12}{24}$ $1364\frac{1}{28}$ $444\frac{17}{77}$ $1009\frac{8}{15}$ $1955\frac{17}{20}$. **18** $11\frac{161}{216}$ $18\frac{665}{2116}$ $13\frac{22}{84}$ $16\frac{173}{192}$ $41\frac{117}{198}$ $103\frac{197}{192}$ $40\frac{97}{120}$. **19** $136\frac{88}{88}$ $444\frac{5}{8}$ $246\frac{21}{35}$ $523\frac{889}{1120}$ $869\frac{12}{32}$ $89\frac{188}{1680}$. **20** $2\frac{33}{56}$ $10\frac{5}{22}$ $13\frac{23}{64}$ $40\frac{5}{16}$ $29\frac{1}{56}$. **21** 90 $105\frac{9}{38}$ $315\frac{15}{32}$ $687\frac{1}{7}$

$82\frac{5}{16}$. **22** $88\frac{3}{64}$ $188\frac{59}{60}$ $13\frac{5}{7}$ $1451\frac{23}{27}$. **23** $65\frac{5}{6}$ 814 2871 $8202\frac{6}{7}$. **24** $73\frac{1}{15}$ $90\frac{9}{14}$ $25\frac{1}{2}$ $157\frac{1}{32}$. **25** $35\frac{79}{551}$ $63\frac{14}{575}$ $93\frac{33}{40}$ $117\frac{171}{187}$.

32 *Answers:* **1** $\$4468\frac{3}{4}$. **2** 345 qt. **3** $2602\frac{7}{8}$ ft. $867\frac{5}{8}$ yd. **4** $\$17.12\frac{15}{16}$. **5** \$508800. **6** $\$64280\frac{2}{3}$, wife $\$24105\frac{1}{4}$, son $\$8035\frac{1}{12}$, hospital. **7** $\$18.28\frac{1}{2}$. **8** $\$8.20\frac{4}{25}$. **9** $\$1057\frac{1}{7}$. **10** $\$1197\frac{8}{11}$. **11** $\$604\frac{4}{9}$. **12** $\$3.06\frac{1}{4}$. **13** $\$313\frac{43}{64}$. **14** $\$14.87\frac{1}{2}$. **15** $\$29.56\frac{1}{4}$. **16** $\$57.49\frac{3}{16}$. **17** \$83.43. **18** $\$54886\frac{6}{11}$. **19** $\$31\frac{23}{36}$. **20** $\$446\frac{1}{5}$. **21** \$867. **22** $\$75.63\frac{17}{36}$. **23** $\$78.72\frac{3}{7}$. **24** $\$15341\frac{3}{8}$. **25** $\frac{13}{5334}$ $\frac{1}{60}$ $3\frac{7}{8}$ $3\frac{51}{80}$ $\frac{1}{8100}$. **26** $\frac{245}{3078}$ $\frac{41}{625}$ $\frac{37}{2835}$ $\frac{4}{27}$ $\frac{1}{8}$. **27** $14\frac{163}{234}$ $\frac{127}{2592}$ $\frac{229}{1080}$ $\frac{5}{2944}$ $\frac{1}{128}$. **28** $\frac{83}{150000}$ $1\frac{1055}{1596}$ $\frac{1}{1920}$ $\frac{1321}{7400}$ $2\frac{2342}{3375}$. **29** 280 408 $8\frac{3}{4}$ $47\frac{5}{9}$ $\frac{211}{2340}$. **30** $16\frac{1}{2}$ 20 $16\frac{61}{90}$ $40\frac{1}{15}$ $873\frac{8}{9}$. **31** $40\frac{2}{3}$ $58\frac{5}{8}$ $158\frac{10}{21}$ $10\frac{36}{47}$ $24\frac{16}{41}$. **32** $2\frac{28}{65}$ $1\frac{145}{247}$ $\frac{157}{475}$ $\frac{35}{122}$ $1\frac{43}{88}$.

Let the pupils be accustomed to multiply by a multiplier placed in any position; thus, in **1**, the solution is written:

$$\begin{array}{rl} 6\frac{7}{8} & \text{Cost of 1 ton.} \\ 650 & \\ \hline 3900 & \\ 568\frac{3}{4} & \\ \hline \$4468\frac{3}{4} & \text{Cost of 650 tons.} \end{array}$$

or, $\$6\frac{7}{8}$, cost of 1 ton, $\times 650 = \$3900$ $+ \$568\frac{3}{4} = \$4468\frac{3}{4}$, cost of 650 tons.

In these solutions the logical multiplier is 650, but the number actually used is $6\frac{7}{8}$. The multiplication should be made with the multiplier in either position.

Cancellation of common factors in both dividend and divisor may be made when convenient; thus, in **25**:

$$6\tfrac{2}{3} \div 400 = \frac{\cancel{20}}{3 \times \underset{20}{\cancel{400}}} = \tfrac{1}{60}.$$

33 *Answers:* **1** $8\frac{5}{6}$ $15\frac{59}{60}$ $5\frac{177}{224}$ $9\frac{4}{39}$ $7\frac{29}{54}$. **2** $8\frac{27}{68}$ $6\frac{257}{270}$ $4\frac{149}{210}$ $6\frac{72}{77}$ $1\frac{199}{270}$. **3** $\frac{67}{198}$ $147\frac{33}{40}$ $4\frac{279}{949}$ $7\frac{5}{369}$ $16\frac{26}{51}$. **4** $23\frac{1}{6}$ $\$.16\frac{23}{104}$ $\$.18\frac{277}{336}$ $.03\frac{227}{308}$ $\$.30\frac{509}{840}$. **5** $\frac{1}{21}$ $\frac{5}{7}$ $2\frac{4}{27}$. **6** $14\frac{12}{17}$ $6\frac{3}{5}$

$\$10\frac{15}{19}$. **7** $\frac{13}{200}$ $7\frac{1}{18}$ $1\frac{15}{16}$. **8** $1\frac{13}{14}$ $1\frac{29}{88}$ $\frac{690}{1001}$. **9** $595\frac{5}{21}$ 108 $75\frac{7}{11}$ 53 $1\frac{91}{810}$. **10** $\frac{53}{300}$ $9\frac{61}{135}$ $3\frac{43}{132}$ $10\frac{38}{147}$ $14\frac{47}{104}$. **11** $9\frac{116}{129}$ $138\frac{22}{29}$ $3\frac{185}{561}$ $95\frac{1}{6}$ $3\frac{117}{161}$. **12** $\frac{18}{23}$ $1\frac{3}{16}$ $1\frac{1}{16}$ $\frac{59}{70}$. **13** $1\frac{191}{214}$ $\frac{49}{69}$ $\frac{2}{43}$ $\frac{47}{135}$. **14** $\frac{588}{3379}$ $\frac{212}{2295}$ $38\frac{4}{5}$ $1\frac{46}{143}$. **15** $2\frac{11}{38}$ $56\frac{9}{31}$ $154\frac{46}{77}$. **16** $15\frac{611}{707}$ $16\frac{113}{118}$ $2686\frac{8}{9}$. **17** $623\frac{1}{2}$ $457\frac{91}{125}$ $2\frac{200}{427}$. **18** 450. **19** $97\frac{1}{2}$. **20** $143\frac{1}{3}$. **21** $1756\frac{4}{11}$. **22** $4\frac{100}{119}$ **23** 275.

34 *Answers:* **1** 125. **2** $\frac{16}{35}$. **3** \$2.40. **4** $22\frac{1}{2}$ bu. **5** $\$.76\frac{4}{5}$. **6** 27. **7** \$1.43 $15\frac{3}{4}$ lb. **8** 20 bu. $\$12\frac{3}{16}$. **9** $25\frac{5}{12}$ bu. apples 60 bu. **10** $222\frac{2}{9}$ 10 suits $4\frac{17}{18}$ yd. **11** $10\frac{10}{27}$ T. $\$30\frac{3}{8}$. **12** $\$5.62\frac{1}{2}$ 200 francs. **13** 6 breadths. 8 breadths. **14** 7 bu. 8 bu. $5\frac{3}{5}$ bu. **15** \$2 gain. **16** $\$\frac{9}{80}$ $\$112\frac{1}{2}$. **17** $283\frac{1}{2}$ gal. $\$66.04\frac{1}{2}$. **18** $\$68\frac{29}{32}$ $22\frac{2}{3}$ da.

The usual forms of oral and written analysis of these concrete problems may be made, and afterwards they may be written out "on a line." As the pupil proceeds with the oral analysis, he writes the number above or below the line; thus, in **10** the pupil may be taught to say: "My answer is to be in yards, so I write 100 yd. as the number to be wrought upon. It will take $\frac{1}{18}$ as many yards to make 1 suit as it will to make 18 suits. I write 18 below the line as divisor. It takes so many yards to make 1 suit. To make 40 suits it will take 40 times as many yards as it takes for 1 suit. I place 40 above the line." The problem when finished appears thus:

$$\frac{100 \text{ yd.} \times \overset{20}{\cancel{40}}}{\underset{9}{\cancel{18}}} = \frac{2000}{9} \text{ yd.} = 222\tfrac{2}{9}.$$

35 Some work with objects may help the pupils to learn this new principle of finding the whole when a part is given. The illustrative work given will show what should be done with the objects. Give the pupils a certain number of counters, as 6, and tell them to show you $\frac{1}{3}$ of the counters; $\frac{2}{3}$ of them; $\frac{3}{3}$ of them. Then give to each pupil 4 counters, and say that they have $\frac{2}{3}$ as many

as you have in your hand. Ask them to point out $\frac{1}{8}$ as many as you have, and then ask them how many more counters they must take to have as many as you. Much illustrative work with dots or marks will help to fix the principle. To be sure that they understand it, give them work of both kinds — a part given, to find the whole, and the whole given, to find a part.

Let the analysis, even of those problems which the pupils can perform orally, be written out in full. It will be seen that the conditions vary considerably and need careful thinking.

36 When any new feature or condition is given, as in **9**, let the pupils try the problem first before attempting to assist, or even before calling their attention to the new condition. If they say, as they will be likely to at first, that 20 gal. is $\frac{3}{8}$ of the whole number, simply ask them how many gallons were left and what part was left; and so lead them to think what part of the whole 20 gal. is equal to.

37 *Answers:* **1** \$.51. **2** $20\frac{1}{3}$ wk. **3** A, \$45 B, \$15. **4** A, \$20 B, \$120. **5** .4 T. .096 T. **6** 32 bbl. **7** 30 peaches. **8** 24 yd. **9** \$17. **10** $\$21\frac{1}{3}$. **11** \$.06 $\$4.52$. **12** $\$.52\frac{1}{2}$. **13** $\$.23\frac{1}{3}$. **14** \$8000. **15** \$200. **16** \$1000. **17** $\$333\frac{1}{3}$. **18** $\$47\frac{17}{23}$.

The pupils should be asked to make written solutions on a line with oral analysis, or to make written analyses with each step indicated and each result designated. These two methods are shown in the following solutions of **11**:

8) \$0.64 cost of $2\frac{2}{3}$ lb.
.08 " " $\frac{1}{8}$ lb.
3
\$0.24 " " 1 lb.
$18\frac{5}{8}$
192
24
20
\$4.52 cost of $18\frac{5}{8}$ lb.

$$\frac{\cancel{\$4}\cancel{0} \times \cancel{3} \times 113}{\cancel{8} \times \cancel{6}_{\cancel{2}}} = \$4.52.$$

Encourage the pupils to perform orally as many of these and the following problems as they can.

38 *Answers:* **1** $\frac{5}{6}$ cd. $1\frac{1}{5}$ da. **2** $2\frac{2}{5}$ da. 2 da. $1\frac{5}{7}$ da. **3** \$.30 \$.36. **4** \$96. **5** \$4.20. **6** \$44000 \$13750. **7** 40 pupils. **8** \$48000 \$36000, widow. **9** \$.34$\frac{2}{7}$. **10** \$.69$\frac{58}{103}$. **11** \$8.40. **12** 6 weeks. **13** $157\frac{17}{19}$ T.

39 *Answers:* **1** $7\frac{3}{11}$ h. **2** $191\frac{1}{5}$ A. **3** \$.80. **4** \$60480. **5** \$16853$\frac{1}{3}$ B, \$8426$\frac{2}{3}$. **6** $154\frac{5}{16}$ lb. \$19.29. **7** $113\frac{3}{4}$ mi. **8** 3 da. **9** 2 da. **10** $\frac{12}{19}$ da. $12\frac{12}{19}$ da. **11** \$750. **12** \$4800. **13** \$140$\frac{7}{8}$. **14** 76 shares \$14.

40 *Answers:* **1** \$753.81$\frac{1}{3}$. **2** \$63$\frac{9}{128}$. **3** \$17$\frac{61}{175}$. **4** \$79$\frac{1}{5}$. **5** \$1579$\frac{3}{4}$. **6** \$2176$\frac{8}{45}$. **7** \$2508. **8** \$86.62$\frac{1}{2}$. **9** House, \$7058$\frac{3}{4}$ Land, \$4235$\frac{1}{4}$. **10** $6\frac{2}{3}$ T. **11** 16 s. ($\frac{1}{8}$ yd. left). **12** \$25116$\frac{2}{3}$. **13** \$15.073+. **14** $\frac{3}{4}$ lb. **15** .07 lb. **16** \$45.26. **17** $27\frac{15}{16}$ jars. **18** $\frac{3}{7}$ $\frac{9}{14}$.

41 *Answers:* **1** $\frac{34}{65}$. **2** \$38.76+. **3** $249\frac{3}{4}$ A. **4** \$48$\frac{6}{7}$. **5** \$1362$\frac{1}{5}$. **6** $66\frac{35}{36}$ yd. **7** \$4.41$\frac{2}{3}$. **8** $37\frac{583}{741}$ h. **9** \$6.50. **10** \$5$\frac{29}{32}$. **11** $42\frac{189}{552}$ h. **12** $162\frac{1}{2}$ bu. **13** \$468$\frac{3}{4}$. **14** 3 yr. 11 mo.+ 2 yr. 4 mo.+ **15** $41\frac{181}{259}$. **16** \$8$\frac{1}{8}$.

42 *Answers:* **1** \$306$\frac{11}{30}$. **2** \$1127$\frac{1}{7}$ gain. **3** \$82.50+. **4** Po., 500 mi.; Kan., 900 mi.; Yel., 1000 mi.; Red., 1200 mi.; Ark., 2000 mi.; St. L., 2200 mi.; Nile, 3300 mi.; Am., 3600 mi. **5** 176 lb. $8\frac{2}{5}$ lb. **6** 20 yr. **7** 100 ft. **8** $32\frac{7}{24}$ gal. $65\frac{1}{48}$ lb. **9** To Sar., 180 mi.; to R., 360 mi.; to B., 480 mi.; to A., 720 mi.; to L., 900 mi.; to J., 1500 mi.; to S. L. C., 1800 mi.; to S. F., 2250 mi.

43 A review of the objective work in decimals to thousandths, given in Book IV., will be found useful in giving a good foundation for the work here given. If the pupils clearly understand the expression and use of decimals to thousandths, there need be no use of objects in dealing with higher denominations. Dwell upon **6** and **7** until they are well understood. If **9** is found too difficult it may be omitted for the present.

44 Lead the pupils to see why the annexing of a cipher at the right of decimals does not change their value.

45 *Answers:* **1** 7576.13558657. **2** \$99.999 \$99.9995. **3** 210.962803. **4** 3.37079 in. **5** 60.8533⅓ rd. **6** 59.0135 A. **7** 390.95347. **8** 652.6939. **9** 837.363. **10** 1.0425 .1953. **11** 280.4274. **12** 9.9191. **13** 1005.991201. **14** 19.9991. **15** 1.94. **16** 114.194. **17** 94.61359. **18** 111.8909. **19** 72.7699. **20** 4.2 .42 .042 7.2 1.62. **21** .03 .64 1.33 .021 6.4. **22** .0101 .1836 .624 .576 .02108.

Some preliminary exercises may be given to show that hundredths of hundredths give ten thousandths, and that thousandths of hundredths give hundred thousandths, etc. This may be shown by common fractions.

It is not necessary after the first few exercises to explain each step in addition and subtraction. Work of this kind in one part of the decimal system ought to be no more difficult than similar work in any other part.

46 *Answers:* **1** 35.77. **2** .3431. **3** 3.9786. **4** 12.353536. **5** 14.404968. **6** 2432.5. **7** 84.78384. **8** 26.3088. **9** 8298.63. **10** 74.2118. **11** 8.64838. **12** 49.0245. **13** 81.2448. **14** 793.074. **15** 3.10156. **16** .0095823. **17** .24609102. **18** .976050. **19** 8476.4072. **20** 67.268. **21** 22.25562. **22** 9.322987. **23** 9247.854. **24** 164.467. **25** 2370.104. **26** 181.80208. **27** 6.0888. **28** 2470.713. **29** 4.8. **30** 1564.9236. **31** .003267 .93654 .0772101 2.228094. **32** 6.1509 .061509 6.09609. **33** .040068 .0941598 2.1005649 .060102. **34** .0144684. **35** 369.84414285. **36** .108 1.08 .01188 .0108108 10.8. **37** 100040. 100040.10004. **38** .6501755 .0650390 .0078520 .262626 .0135395. **39** 2050.3935. **40** 803.8024. **41** 3.5000056. **42** 10000.84. **43** .087675. **44** 10924.914. **45** 1312.6476. **46** 44850.3102. **47** 616.9472. **48** 684.39378. **49** 61.2612. **50** 160160. **51** .8757567. **52** .203775. **53** .58313850. **54** 520.96.

47 *Answers:* **1** 88.787. **2** 94.261544. **3** 1424.08476. **4** 878.17221. **5** 296.478. **6** 525.5491. **7** 7852.547.

8 214.9854. **9** 7871.0072. **10** 52413.821. **11** 2551.6278. **12** 104066.5212. **13** 633.8787. **14** 1052.62201. **15** .1984512. **16** 32.1082179. **17** 981. **18** 7006.3. **19** 109.46. **20** 518.1357. **21** 488.08032. **22** 3793.9408. **23** 2593.44. **24** 80012.6. **25** 1738.8 yd. **26** \$8154.675. **27** 1153.705 mi. **28** \$112.4475. **29** \$1038.744. **30** \$37.0476. **31** \$34.832. **32** \$2044.80. **33** \$3548.12. **34** \$35574.59435. **35** .2 .02 .004 .4 3.4 .00004 3.334 .00034 .0000004. **36** 20 200 20000 .2 .02 300000 .003 3000000 3000. **37** .006 .6 10 .01 .001 .0001 1 .1 9.6.

Before the exercises in division of decimals are begun, there should be much practice upon such work as the following: "Tenths of hundredths give what? hundredths of tenths? hundredths of hundredths? tenths of thousandths? thousandths of hundredths? thousandths of thousandths? tenths of tens? hundredths of tens? tenths of hundreds? hundredths of hundreds? etc. Tenths divided by tenths give what? hundredths by hundredths? units by tenths? tenths by hundredths? hundredths by tenths? thousandths by hundredths? thousandths by tenths?" etc.

In division of decimals there are three possible cases, viz.: (1) Division in which the divisor and dividend are of the same denomination; *e.g.* $.6 \div .3$. (2) Division by a decimal in which the divisor is of a lower denomination than the dividend; *e.g.* $.8 \div .2$; $.8 \div .04$. (3) Division by a decimal in which the divisor is of a higher denomination than the dividend; *e.g.* $.04 \div .2$; $.008 \div .04$. The first of these cases ought to give no trouble to the pupils. It is readily seen that 4 tenths will be found as many times in 8 tenths as 4 quarts in 8 quarts, etc. That is, they see that when the divisor and dividend are of the same denomination the quotient is a whole number (of times). Neither will there be difficulty in dividing by a whole number. The pupils see at once that $.08 \div 4$ is $\frac{1}{4}$ of 8 hundredths $= 2$ hundredths, which is the same denomination as the dividend. All other classes of problems in division may be easily resolved into one or the other of the classes already understood. For example, if it is required to find

how many times 4 hundredths is contained in 2 tenths, the operation would be expressed, $.2 \div .04$, or $.04\overline{).2}$. The pupils know that a cipher or ciphers placed at the right of decimals do not change their value, and therefore express the operation thus: $\overset{.04)}{5}\underline{.20}$. All such problems may be performed in a similar way. Again, if it is required to find what part of .2 is .04, the operation would be expressed, $.04 \div .2$, or $.2\overline{).04}$. The pupils should know, that when the divisor and dividend are multiplied or divided by the same number, the quotient is the same. Therefore, by multiplying the divisor and dividend in this problem by ten, the operation may be expressed thus: $\overset{\times 2.)}{.2}\underline{\times 0.4}$. Really, the pupils should be sufficiently familiar with multiplication in the various denominations to know that, when the dividend is hundredths and the divisor is tenths the quotient must be tenths, or when the dividend is thousandths and the divisor is tenths the quotient must be hundredths, etc.

After several problems have been done in this way the pupils will see that they can point off as many decimal places in the quotient as the number of decimal places in the dividend exceeds the number of decimal places in the divisor.

48 *Answers:* **1** *a* .8002; *b* 32000; *c* 834400; *d* 3; *e* .9. **2** *a* 3000.375; *b* 1066.666+; *c* .0754+; *d* .00301; *e* .00008. **3** *a* 1066.666+; *b* 26900; *c* 6680; *d* 255; *e* .0433+. **4** *a* 116.5263; *b* 47.946; *c* 2.08; *d* .2726+; *e* .084. **5** *a* 1.3333+; *b* 16000; *c* 1600; *d* 82000; *e* 146.3902+. **6** *a* 9050; *b* 960; *c* .0463; *d* 9.26; *e* 1027.7279+. **7** *a* .8; *b* .08; *c* 80; *d* 800; *e* 8000. **8** *a* 1402.5; *b* 140.25; *c* 140.25; *d* 7.0125; *e* 140.25. **9** *a* 2090; *b* .02; *c* 2000; *d* 4.06; *e* .5236. **10** 402.3739. **11** 200170. **12** 48.63794+. **13** .0092. **14** 2980. **15** *a* 1.4634+; *b* .0047+; *c* 499.5004+; *d* .7976; *e* 14.1843; *f* 50000; *g* 133.3333+. **16** *a* 54.225+; *b* 13.8910+; *c* 548.5301; *d* 196176.470+; *e* 496761.904+; *f* 30.2641; *g* 333.3333+. **17** *a* 4073; *b* 27900+; *c* 17.2558+; *d* 90.065; *e* 9008; *f* 344.5783. *g* .1568+.

49 *Answers:* **1** *a* .4673; *b* 2.0925; *c* .00007+; *d* 502588.94+; *e* 8.014. **2** *a* .0000686+; *b* 59.0277+; *c* 49.975+; *d* 1795.537+; *e* 8612.8345+. **3** *a* 2.2168+; *b* 63.8297+; *c* .5602+; *d* .00541+; *e* 4664.067+. **4** 100 yd. 1000 yd. 110 yd. **5** 12000 apples. **6** 20 yd. 200 yd. 250 yd. **7** 12 qt. $19\frac{2}{7}$ qt. 32 qt. 2000 qt. **8** 7 da. 92 da. 800 da. **9** \$4.30. **10** \$4.20. **11** 8.2+ h. **12** 44.6 mi. **13** 650 A. **14** 90 pr. 800 pr. **15** 240 coats. **16** 23 da. 605.7+ da. **17** 25 cd. **18** \$1.50 \$150.75.

50 *Answers:* **1** 120 casks. **2** 50900 yd. **3** 5.84. **4** 504.6 boxes. **5** 42.5 A. **6** 87.9 A. **7** .5 .75 .125 .625 .875 .375. **8** .05 .075 .08 .0625 .075 .15. **9** .15 .16 .38095+ .1875 .9375 .4218+. **10** 1.4 $1.16\frac{2}{3}$ 1.8 2.025 1.5625. **11** 1.75 2.875 14.7619+ 21.9375 40.015. **12** 15.3125 20.4 15.53125 27.1875. **13** $\frac{1}{2}$ $\frac{21}{50}$ $\frac{1}{125}$ $\frac{3}{125}$ $\frac{7}{50}$ $\frac{1}{400}$ $19\frac{1}{8}$. **14** $14\frac{3}{180}$ $\frac{547}{50000}$ $4\frac{243}{4000}$ $\frac{2561}{250000}$. **15** $304\frac{253}{50000}$ $3\frac{4457}{50000}$ $14\frac{1}{400}$ $\frac{1001}{200000}$. **16** $\frac{1}{80}$ $\frac{1}{500}$ $1\frac{3}{40}$ $\frac{8}{800}$ $\frac{1}{2000}$ $\frac{401}{50000}$. **17** $\frac{1}{8}$ $\frac{1}{30}$ $\frac{1}{15}$ $\frac{2}{3}$ $\frac{1}{800}$ $\frac{1}{180}$. **18** $\frac{7}{8000}$ $3\frac{23}{330}$ $\frac{3}{800}$ $\frac{33}{400}$ $1\frac{1}{4}$ $\frac{1}{180}$. **19** 2.35. **20** 7.918. **21** 10.5785. **22** \$1.728. **23** $\$8\frac{3}{4}$ \$110.83+. **24** 14.6 yd.

In changing common fractions to decimals lead the pupils to think of the fraction as another form of expression for division; thus, $\frac{3}{4} = 3 \div 4$ or $\frac{1}{4}$ of 3.

51 *Answers:* **1** 9.6875 yd. **2** 80 sq. rd. 40 sq. rd. 60 sq. rd. 100 sq. rd. **3** 38 A. 94 sq. rd. 18 sq. yd. 1 sq. ft. 50.4 sq. in. 9 A. 128 sq. rd. 150 A. 112 sq. rd. 4 A. 126 sq. rd. 12 sq. yd. $\frac{9}{10}$ sq. ft. 7 sq. yd. 6 sq. ft. 100.2 sq. in. **4** 2.0225. **5** 52.545 mi. **6** \$565.88 \$126. **7** \$263.4375. **8** 67 lb. 1 oz. 12 pwt. 12 gr. **9** 5017.95 lb. **10** 15. **11** 2.863+ oz. **12** $\frac{1}{8}$ $\frac{2}{3}$ $\frac{14}{33}$ $\frac{4}{37}$ $\frac{16}{999}$. **13** $.\dot{3}$ $.\dot{1}42857\dot{1}$ $.1\dot{6}$ $.0\dot{9}$ $.2916\dot{6}$. **14** $1\frac{7}{15}$ $\frac{1}{30}$ $11\frac{3}{100}$ $\frac{1}{60}$ $1\frac{2}{33}$. **15** $\frac{1}{15}$ $\frac{1}{30}$ $\frac{1}{9990}$.

In teaching repetends let the pupils see by examples that the figures of a repetend express the numerator of a common fraction having as many nines in the denominator as there are figures in the repetend; thus, $.1\dot{6} = .1\frac{4}{9}$, and $.\dot{1}0\dot{8} = \frac{107}{999}$.

52 *Answers:*

	2	3	4	5	6	7
1	460.08	.75	$\frac{7}{200}$	4.115	.785	4.83
2	80.0075	.6666	$\frac{3}{500}$	6.009	.6726	6.6696
3	708.03006	.375	$\frac{7}{200}$	180.035	.41	180.375
4	.00875	.3125	$\frac{33}{40}$	6.861	1.1375	6.3485
5	80.000076	.5555	$20\frac{103}{500}$	20.2867	20.7615	.6362
6	.0098	.5833	$8\frac{1}{25}$	88.083	8.6233	80.6263
7	.0000708	.7333	$\frac{39}{500}$	3.0789	.8113	3.7342
8	.000604	.3895	$\frac{21}{2000}$	700.8105	.4	701.1895
9	.190	.7083	$48\frac{3}{50}$	62.478	48.7683	15.1263
10	450.000087	.6333	$8\frac{19}{250}$	8.15676	8.7093	.71409
11	.10050	.425	$\frac{1}{1250}$	3.0218	.4258	3.446
12	70.070	.18	$\frac{141}{200}$	1.707	.885	1.182
13	.545	.3166	$800\frac{3}{5}$	800.6075	800.9166	.3241
14	800.07	.2	$70\frac{1}{125}$	70.215	70.208	.407
15	1004.009	.0583	$900\frac{29}{50}$	930.67	900.6383	30.1483
16	350000.4	.0622	$70\frac{1}{200}$	74.013	70.0672	4.0702
17	.00070	.0045	$1\frac{3}{5000}$	1.2415	1.0051	.2454
18	20.000010	.127	$8\frac{1}{25}$	8.064	8.167	.151
19	48.070	.059	$\frac{3803}{5000}$	8.7507	.7802	8.0897
20	700.50	.001	$1\frac{1}{125}$	101.800	1.009	100.801

	8	9	10	11
1	460.115	464.16	460.83	4.045
2	80.0135	86.0105	80.6741	5.997
3	708.06506	888.03006	708.40506	179.965
4	.83375	6.04475	.32125	5.211
5	100.206076	80.080776	80.55576	20.1253
6	8.0498	80.0528	.5931	72.003
7	.0780708	3.0009708	.7333708	2.9229
8	.011104	700.800604	.390104	700.7895
9	48.25	14.608	.8983	33.642
10	458.076087	450.080847	450.633387	7.99524
11	.1013	3.1215	.5255	3.0202
12	70.775	71.072	70.25	.297
13	801.145	.5525	.8616	800.5925
14	870.078	800.277	800.27	69.801
15	1904.589	1034.099	1004.0673	870.49
16	350070.405	350004.408	350000.4622	65.997
17	1.0013	.2416	.0052	.7597
18	28.04001	20.02401	20.12701	8.016
19	48.7906	56.1001	48.1296	7.3095
20	701.508	801.30	700.501	99.792

	12	13	14	15	16
1	460.045	456.	.35	35.	73.44
2	80.0015	74.0045	.06	6.	108.054
3	707.99506	528.03006	.35	35.	3240.
4	.81625	6.02725	8.25	825.	108.648
5	59.794076	79.919376	202.06	20206.	1.4526
6	8.0302	80.0332	80.4	8040.	1440.774
7	.0779292	3.0008292	.78	78.	54.0162
8	.009896	700.799396	.105	10.5	12614.4
9	47.87	14.228	480.6	48060.	259.524
10	441.924087	449.919327	80.76	8076.	1.45368
11	.0997	2.9205	.008	.8	54.378
12	69.365	69.068	7.05	705.	18.036
13	800.055	.5375	8006.	800600.	.135
14	730.062	799.863	700.08	70008.	3.726
15	103.429	973.919	9005.8	900580.	541.62
16	349930.395	349996.392	700.05	70005.	72.144
17	.9999	.2402	10.006	1000.6	4.3362
18	11.96001	19.97601	80.40	8040.	.432
19	47.3494	40.0399	7.206	720.6	144.5418
20	699.492	599.7	10.08	1008.	1814.40

	17	18	19
1	4.6008	50.14872	.1428
2	.800075	8.7208175	.036018
2	7.0803006	77.17527654	6.3
4	.0000875	.00095375	4.9797
5	.80000076	8.720008284	1.6306242
6	.000098	.0010682	643.54572
7	.000000708	.0000077172	.2340702
8	.00000604	.000065836	7.3584
9	.00190	20.71	692.92908
10	4.50000087	49.050009483	.65221776
11	.0010050	.0109545	.0024168
12	.70070	7.63763	.70641
13	.00545	.059405	6.0045
14	8.0007	87.20763	14.491656
15	10.04009	109.436981	27098.4522
16	3500.004	38150.0436	280.58004
17	.0000007	.00000763	.24104454
18	.20000010	2.18000109	.19296
19	.48070	5.23963	5.78649006
20	7.005	76.354	101.6064

	20	21	22	23
1	16.10280	.0035	40.8	4.6008
2	.480045	.0006	60.03	.800075
3	24.7810521	.0035	1800.	7.0803006
4	.00721875	.0825	60.36	.0000875
5	1616.049535656	2.0206	.807	.80000076
6	.078792	.804	800.43	.000098
7	.0000055224	.0078	30.009	.000000708
8	.000006342	.00105	7008.	.00000604
9	9.1314	4.806	144.18	.0019
10	3634.200702612	.8076	.8076	4.50000087
11	.0000804	.00008	30.21	.001005
12	49.39935	.0705	10.02	.7007
13	436.327	80.06	.075	.00545
14	56011.30056	7.0008	2.07	8.0007
15	904190.46522	90.058	300.9	10.04009
16	24501778.002	7.0005	40.08	3500.004
17	.00070042	.10006	2.409	.000007
18	160.8000804	.8040	.24	.2000001
19	34.639242	.07206	80.301	.4807
20	706.104	.1008	1008.	7.005

	24	25	26	27
1	460080.	136.	70.	116.5714+
2	80007.5	200.1	12.	1000.5
3	708030.06	6000.	70.	5142.8514+
4	8.75	201.2	1650.	7.3163+
5	80000.076	2.69	40412.	.0039+
6	9.8	2668.1	16080.	9.9555+
7	.0708	100.03	156.	38.4730+
8	.604	23360.	21.	66742.857+
9	190.	480.6	96120.	.3
10	450000.087	2.692	16152.	.01
11	100.5	100.7	1.6	3776.25
12	70070.	33.4	1410.	1.4212+
13	545.	.25	1601200.	.000009+
14	800070.	6.9	140016.	.0029+
15	1004009.	1003.	1801160.	.0334+
16	350000400.	133.6	140010.	.0572+
17	.7	8.03	2001.2	.2407+
18	20000.01	.8	16080.	.0029+
19	48070.	267.67	1441.2	11.1436+
20	700500.	3360.	2016.	100.

53 *Answers:* **1** 1449.0129. **2** 5.049150. **3** 141.285 A. **4** 9.85. **5** $21\frac{1}{2}$ yd. $9\frac{1}{2}$ yd. $84\frac{7}{10}$ yd. 20 yd. **6** $17\frac{1}{2}$ $108\frac{7}{12}$ \$7.368. **7** 28.7 T. **8** \$84.69+ \$415.181 \$16988.81. **9** 16.4008324. **10** \$1135.3375. **11** 33.4 da. **12** \$30483.60 \$17419.20. **13** \$46.426 \$1.946. **14** 65.78+ da. **15** \$5404.20. **16** 26666.66+ sq. ft. 120000 sq. ft.

54 *Answers:* **1** 4.11375. **2** 48. **3** \$14.748+. **4** $1\frac{29}{210}$ bbl. **5** \$7.3125. **6** \$.0002 250000. **7** 1466.542 bu. **8** 21.825. **9** \$42.59025. **10** \$5.405+. **11** \$1.53. **12** 357.88839. **13** \$27.133+. **14** 697. **15** 1366.326 bbl. **16** $9\frac{3}{8}$ mi. 3000 mi.

55 *Answers:* **4** $19541\frac{1}{2}$ ft. **5** 102 ft. 1224 in. **9** 15 yd. 2 in. **10** 1 mi. 4720 ft. **11** 2 mi. 269 rd. 5 yd. **12** $\frac{1}{210}$ $\frac{1}{10}$. **19** 3732 in. **20** 2654 ft. **21** 10989 ft. **22** 1 mi. 280 rd. $1\frac{7}{8}$ mi.

The pupils are supposed to have had some practice in reduction before beginning this section. (See Section VI., Book IV.) Many of these problems should be performed orally, but the solutions may be written out in full for the sake of learning a good form of written work.

56 *Answers:* **1** $5\frac{5}{8}$ mi. **2** 264 paces 2112 paces. **3** 30.6 mi. **4** 1485 paces. **5** $198\frac{2}{11}$ rd. **6** 1 mi. 208 rd. 2 yd. 2 ft. **7** $208\frac{8}{19}$ times. **8** 15 rd. 2 yd. 4 ft. \$5.87$\frac{1}{2}$. **9** \$1.947. **10** \$12. **11** 66 ft. 792 in. **12** 7.92 in.

57 *Answers:* **1** 396 in. 617.76 in. **2** 5 ch. 80 ch. **3** 3659.04 in. 304.92 ft. **4** 498 ch. 32868 ft. **5** 83952 in. **6** 70604.88 in. **7** 2 li. 4.16 in. 6 li. 2.48 in. **8** 17 li. 5.36 in. 75 li. 6 in. **9** 2 ch. 40 li. 6 ch. 80 li. **10** 16 ch. 84 li. **11** 1 ch. 7 li. 2.56 in. 1 ch. 26 li. .08 in. **12** 16 ch. 50 li. **13** 23 li. 7.84 in. 2 ch. 27 li. 2.16 in. **14** 1 ch. **15** 12 ch. **16** 541.2 ft. **17** 1056 ft. **18** 2811.6 ft. 53 mi. $3\frac{1}{13}$ min. **19** $2\frac{2}{3}$ min. .35+ mi.

Measurements by surveyor's measure are not generally made now, and the problems here given may be omitted if thought best.

Exercises with the surveyor's measuring tape may be substituted. On this tape are given the feet, and tenths and hundredths of feet.

Pupils are supposed to have been taught angle, triangle, and rectangle, and to have had some practice in finding areas. (See pages 84–86, Book IV.) Some of the descriptions here called for, however, are new, and the figures may have to be taught again. Let the descriptions be made by the pupils, and let them be made from the objects or illustrations. For example, the rectangle may be shown, and attention be called to the number of sides of the figure and to the kind and number of angles. From these facts the pupils are led to say that "a rectangle is a four-sided figure having four right angles."

58 In explaining the process of finding the area of a rectangle the pupils should be led to multiply the number of square units in a row by the number of rows, as previously shown. (See Manual, pp. 49, 75.)

59 *Answers:* **1** 10 A. **2** 1 A. 140 sq. rd. **3** 3 A. 93$\frac{1}{3}$ sq. rd. **4** 106$\frac{2}{3}$ sq. rd. 3226$\frac{2}{3}$ sq. yd. **5** 104544 sq. ft. **6** 585446.4 sq. ft. 13.44 A. **7** 1424025.1872 sq. ft. 32.69 A. **8** 4 yd. 0 ft. **9** 45.375 ft. **10** 21$\frac{2}{5}$ yd. **11** 7$\frac{7}{25}$ in. **12** \$762.30. **13** \$64. **14** 642$\frac{2}{3}$ sq. yd. 112$\frac{1}{2}$ yd. 241 sq. ft. **15** 21$\frac{3}{5}$ yd. 2$\frac{3}{4}$ rolls. **16** \$38.25+. **17** 11 yd. **18** \$846.28+.

If the pupils find difficulty with exercises in which the area and one dimension are given to find the other, lead them to see the process from simple illustrations. (See page 83, Book IV.) The drawing of plans in illustration of problems should be required when needed.

60 *Answers:* **8** 432 sq. ft. **9** 5400 sq. ft. **10** 6 rd. **11** 10 ft. **12** 8712 ft. **13** 4356 sq. ft. **14** 10. **15** $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{33}$. **16** $\frac{4}{10}$ $\frac{24}{88}$ $\frac{108}{132}$. **17** .25 .01875 .058$\frac{1}{3}$. **18** .0075+ .1628+ .339+. **19** $\frac{1}{4}$ $\frac{1}{32}$ 1$\frac{1}{8}$. **20** .00625 .875. **21** 1.8.

The terms horizontal and vertical as applied to lines should be described as having a certain direction. Comparisons with the

surface of still water and with a plumb line will lead to a good description of horizontal and vertical lines. The figures given should be recognized as quadrilaterals, and the reason why they are quadrilaterals should be given. Parallel lines and parallelograms, which have been described in connection with exercises in Book IV., should be described again.

The descriptions called for in 4 may be given, if thought advisable, in the form of a definition. In teaching a definition the object or process to be defined should be brought into the presence of the pupils, and by questions they should be led to observe those features of the object or process which must be named in the definition. For example, in teaching the definition of a square, a square surface, preferably of a cube, is presented. The pupils are led to see that it is a *plane figure* (that having been previously taught), that it is *bounded by four straight lines*, that *the sides are equal*, and that it *has four right angles*. From these facts the following statement may be drawn from the pupils: "A square is a plane figure which is bounded by four equal straight lines, and which has four right angles." If a rectangle has been already defined, the pupils might say that the square is a rectangle whose sides are equal. Whenever the language of a pupil's definition is faulty, lead him to see the faulty or bungling construction of the definition by questions or by comparison with a correct form.

The following definitions may aid teachers in drawing from the pupils accurate statements:

A rhombus is an oblique-angled parallelogram whose sides are equal.

A rhomboid is an oblique-angled parallelogram whose opposite sides only are equal.

A trapezoid is a quadrilateral which has only two sides parallel.

A trapezium is a quadrilateral which has no two sides parallel.

61 *Answers:* **1** 320 sq. ft. 90 sq. ft. $101\frac{1}{8}$ sq. ft. 3 yd. 4 in. $42\frac{5}{14}$ sq. yd. $83\frac{1}{8}$ sq. ft. $123\frac{1}{8}$ sq. rd. 6 A. 40 sq. rd. 2 A. 137 sq. rd. $189\frac{3}{4}$ sq. ft. 40 A. 2 A. 130 sq. rd. $3\frac{17}{21}$ rd. $24\frac{8}{33}$ rd.

19½ rd. 5542922.88 sq. ft. **2** $25 $60 $156.25. **3** $378. **4** $1158.75. **5** $7224.35. **6** $.005. **7** $1155.75 gain.

5 may be performed by finding first the square chains, and then dividing by 10, the number of square chains in 1 A. Performed on a line, the solution would be :

$$\$240 \times \left(\frac{68.18 \times 44.15}{10}\right)$$

62 *Answers:* **1** 25 sq. yd. 27 yd. **2** 10⅔ sq. yd. 9⅓ sq. yd. **3** 30 rd. **4** 10 ft. **5** 29⅔ yd. **6** 7$\frac{17}{25}$ sq. ft. **7** 36¾ sq. ft. **8** 933⅓. $11.66⅔. **9** 30. 145½. **10** $180 cheaper by sq. ft. **11** $5.31. **12** 3 10 6 20. **13** 20 sq. yd. **14** 39.98+ A. **15** $35.50.

Nearly all of these problems should be illustrated by plans. In **1**, assume that the width of the carpet is 1 yd. or ⅔ yd. In **2**, let the pupils find for themselves how to make allowance for corners. A properly-drawn plan will give all needed assistance. The bricks mentioned in **12** are of common size.

63 *Answers:* **1** 180 sq. ft. **2** 360 sq. ft. 2640 sq. ft. **4** $2450.25. **5** $217.80. **8** 2176 sq. ft. **9** 4416 sq. ft.

Finding the area of the figure in **1** is a review of what has been taught previously, but the hint of a dotted line may have to be given before the pupils can find the correct area. Other exercises similar to this may be useful for practice. The pupils will discover, when they compare answers to **4**, that the shape of the lot, as well as the size, determines the length of the boundary line. To solve **6**, the given polygon must be divided into triangles or rectangles, and the needed dimensions be determined by measurement. In **7**, let the pupils estimate the dimensions of the figures not closer than the sixteenth of an inch.

64 A teaching lesson from the cube should precede **1**. The following facts should be observed and stated by the pupils : "The cube is a solid. (They are told that anything which has length, breadth, and thickness is a solid.) It is bounded by six faces. The

faces are equal to each other. Each face is in the form of a square." All these facts may be gathered into one statement forming a definition, thus : A cube is a solid bounded by six equal square faces.

The blocks used in **2** should be 1-inch cubes. The work indicated in **3** should be performed objectively with the inch cubes. The rows, layers, and number of layers should be observed by the pupils, and the number of cubic inches found by multiplying the number of cubic inches in a row by the number of rows, and this product by the number of layers. This should be repeated until it is well understood.

65 *Answers:* **1** 60 cu. in. **2** 1728 cu. in. **3** 41472 cu. in. **4** 20736 cu. in. **5** 4536 cu. in. 2.6 cu. ft. **6** 4096 cu. in. **7** 72 cu. ft. **8** 169 cu. ft. **9** 27 cu. ft. **10** 8 cu. yd. 20 cu. yd. 3.51 cu. yd. **11** 14 cu. yd. 22 cu. ft. 23 cu. yd. 19 cu. ft. **12** 2 cu. ft. 544 cu. in. 6 cu. ft. 1712 cu. in. **13** 1152 cu. in. 1296 cu. in. 23328 cu. in. **14** $\frac{1}{12}$ $\frac{8}{8}$. **15** 7 in. by 5 in. by 3 in. 105 cu. in. **16** 208 sq. in. 142 sq. in.

There are two ways of expressing the operation of changing a number from one denomination to another. Thus, in **10**, the pupil may be led to say: "There is $\frac{1}{3}$ as many cu. yd. as there are cu. ft."; or, "There are as many cu. yd. in 216 cu. ft. as there are nines in 216"; or, "as many cu. yd. as 9 cu. ft. is contained times in 216 cu. ft." But in all such explanations care should be taken that the pupil sees the reason for the step taken. Sometimes it is well in such exercises to ask if the answer is to be less or more than the given number, and then to ask what part as many or how many times as many.

66 *Answers:* **1** 128 cu. ft. = 1 cd. 8 cd. ft. = 1 cd. 16 cu. ft. = 1 cd. ft. **3** 64 cu. ft. 48 cu. ft. 96 cu. ft. **4** 80 cu. ft. 56 cu. ft. 88 cu. ft. **5** 4 cd. ft. $6\frac{1}{4}$ cd. ft. 24 cd. ft. **6** $\frac{1}{4}$ $\frac{7}{16}$ $\frac{3}{4}$. **7** $1\frac{1}{4}$ cd. $1\frac{9}{16}$ $3\frac{3}{4}$. **8** \$3.12$\frac{1}{2}$. **9** 396 cu. ft. $24\frac{3}{4}$ cd. ft. $3\frac{3}{32}$ cd. **10** \$40$\frac{1}{2}$. **11** \$136.64. **12** 1248 packages. **13** 2027.5 papers.

To give a clearer impression of a cord of wood, splints 4 in. long might be piled between stakes. The pile should be 8 in. long and 4 in. high. In the same way 1 cord foot could be shown. Piles of other dimensions could be made for measurement. The pupils will find this interesting as well as profitable work.

67 *Answers:* **1** 227 p. 9051 p. 617 p. **2** 30 sq. rds. **3** 6 strips. **4** \$76.56. **5** $102\frac{3}{32}$ ft. $544\frac{1}{2}$ ft. **6** 150 ft. $22\frac{1}{16}\frac{4}{5}$ sq. rds. **7** \$24.80. **8** 60 posts 1530 pickets. **9** $\$13.22\frac{2}{3}$ $529\frac{2}{3}$ sq. ft. \$1.68. **10** $198\frac{1}{8}$ bu. **11** $1770\frac{1}{4}$ sq. ft. **12** $\$6.52\frac{1}{2}$.

In the first part of **1**, if the pickets are flush with the ends of the fence, one space will be only $\frac{1}{2}$ in. wide. Let the pupils see this by dividing the space in inches between one end of the fence and the inside edge of the picket on the other end by $3\frac{1}{2}$, the number of inches that one picket and one space occupy. Make no account of corners in the last part of 1.

68 *Answers:* **1** $252\frac{2}{1}\frac{8}{3}\frac{8}{5}$ cu. ft. **2** $\$3\frac{1}{3}\frac{8}{3}$, or \$3.878 +. **3** $5\frac{2}{3}\frac{4}{2}$ cd. **4** $30\frac{2}{4}\frac{0}{3}\frac{8}{5}\frac{8}{8}\frac{8}{0}$. **5** $4\frac{8}{2}\frac{9}{5}\frac{9}{0}$; $16\frac{2}{3}\frac{3}{7}\frac{4}{1}$. **6** $21\frac{1}{8}$ cu. ft. **7** $32\frac{1}{1}\frac{1}{8}$ tons $\$176.34\frac{8}{8}$.

The making and solving of original problems in which the pupils' own measurements are given will be found especially valuable. Lead the pupils in such work to apply all the principles that they have learned.

69–70 For information as to U.S. coins see Book IV., page 89, and note thereon in the Manual. Most of the problems on these pages should be solved orally.

71 *Answers:* **1** $\$\frac{4}{5}$. **2** \$1.48 +. **3** \$117. **4** $\$78.62\frac{1}{2}$. **5** \$233.0625. **6** \$123.12 gain. **7** $9\frac{3}{8}$ bu. 50 days. **8** \$2.46. **9** \$51.84. **10** Brown, \$10.50 Eaton, \$8.79 Black, \$8.925 Fuller, \$8.32 O'Connell, \$10.00 T. Smith, \$10.74 A. Smith, \$11.42 Hall, \$11.70 Woods, \$11.375 Kelley, \$11.81 Reed, \$8.49.

Lead the pupils to employ shorter processes when they are easily understood; thus, in **1**, by reducing 15 lb. and $6\frac{2}{3}$ lb. to thirds the pupil can be led to see that $\frac{2}{3}$ will cost $\frac{1}{3}$ as much as $\frac{4}{3}$. $\frac{1}{3}$ of a dollar $= 44\frac{4}{9}$¢. The pupils should be told that generally the fraction of a cent is dropped if it is less than half, but that in this and all similar cases the extra cent would probably be charged. Fractions should be used whenever the process is shortened thereby; thus, in **3** and **5**: $6\frac{1}{2}$ M. and $28\frac{1}{4}$ M. should be used instead of 6500 and 28250.

72 *Answers:* **1** $8.50 $4.25 $1.70 $6.38 $15.86. **2** $62.50 $625 $312.50 $156.25. **3** $67\frac{1}{2}$¢. **4** 48¢ $38.40 $28.80. **5** $1.50 $4.50 $5.625. **6** $175 $35 $140 $131.25. **7** $3.60 $360 $180 $270. **8** $7.68 gain. **9** $4.20 gain. **10** $3\frac{3}{8}$. **11** $19\frac{1}{2}$. **12** $20.34. **13** $3\frac{1}{8}$ lb. $274\frac{1}{2}$ qt. **14** 10.46 lb. **15** $\frac{1}{11}\frac{4}{3}$. **16** 223.25 dollars. **17** 5 lb. $8\frac{3}{4}$ oz.

Most of these exercises can be performed orally. Lead the pupils to work through 100 or 10 when more convenient, as in **1** to **7**.

73 *Answers:* **1** 20. **2** 1 lb. $4\frac{3}{4}$ oz. **3** 144 2976 6096. **4** $31\frac{1}{8}$ spoons. **5** $57\frac{3}{4}$ cu. in. **6** 11.63 qt. **7** 2150.4 cu. in. 537.6 cu. in. 14515.2 cu. in. 6182.4 cu. in. **8** 17.14 qt. **9** 348.348 bu. **10** 598.4 gal. 3111.9 gal. 72.3 gal. **11** 18977.14 gal. **12** 1386 cu. in. larger. **13** 37.23 qt. **14** 1.6 qt.

74 *Answers:* **5** 72 sq. rd. 12.8 sq. rd. 5.6 sq. rd. 261.33 sq. ft. 310 sq. rd. **11** 4 rd. 5 yd. 3 rd. 1 ft. 6 in. 17 yd. $512\frac{2}{3}\frac{3}{3}$ rd. **12** 6 sq. ft. 23 sq. yd. 8 sq. rd. **18** .25 rd. .4 rd. **19** .0625 mi. .027 + mi. **20** .1 A. .0037 + A. **29** .459 T. 13600 lb.

Have as many of these performed orally as possible. A few whose answers are not given may have to be performed by the aid of figures. For the sake of practice in written analysis some of the easier solutions may be written out in full, as, for example:

4	
.8	sq. rd.
30¼	
2.42	sq. yd.
9	
3.78	sq. ft.
144	
1152	
1008	
112.32	sq. in.

Ans.: 2 sq. yd. 3 sq. ft. 112.32 sq. in.

9
60)6200 sec.
60)103 min. 20 sec.
1 h. 43 min.
Ans.: 1 h. 43 min. 20 sec.

75 Most of these exercises can be performed orally, advantage being taken of convenient fractions whenever possible.

Frequent practice should be had in writing receipts and bills.

76 *Answers:* **2** Balance, $31.43. **3** Balance, $89.25.

Ask the pupils to explain each item of these accounts, and lead them to distinguish readily which items belong to the debit side and which to the credit side of the account.

Encourage them to keep a cash account of their own in the form given, and to balance the account at the end of every week.

77 *Answers:* **1** Balance, $1063.45. **2** Balance, $91.36. **3** Balance, $11.40.

78 Point out the difference of form in these two bills of sale, and show the use of the term *debtor* (Dr.) in the second bill. Blank forms are easily obtained, and yet it would be well for the pupils to be able at any time to rule their own forms.

79 *Answers:* **1** Balance, $166.07. **2** Balance, $4.87½. **3** Balance, $28.705. **4** Balance due June 1, $23.24.

Explain fully the use of the term *creditor* (Cr.). Show that a man is debtor for all that he receives, and that he is creditor for all that he does or pays out. Give several exercises illustrating this point.

80 *Answers:* **1** Balance due, \$44.27. **2** Balance due, \$137.35. **3** S. L. Childs owes Asa Howland, \$109.12. **4** \$15.75, one man; \$1890. **5** \$7.90.

In the last answer to **4** the assumption is that 120 men are employed the entire time given, with no lost time.

81 *Answers:* **1** \$15465.05. **2** \$20208.14. **3** \$14379.90. **4** \$12300.02. **5** 11600 25200 696000 375000. **6** 63000.

5 to **10** should be performed orally, and as rapidly as possible. It may be well to have the pupils go over these exercises two or three times for the sake of acquiring facility in multiplying or dividing by the fractional parts of 10 and 100

82 *Answers:* **20** \$130.26. **21** \$115.60. **22** \$559.24. **23** \$297.15. **24** \$165.84. **25** \$249.84. **26** \$9.55. **27** \$108.54.

The first 19 exercises are for quick oral practice. In multiplying by a number two or three units less than 100 or 1000, lead the pupils to make two multiplications and subtract; thus, in **4**: $93 \times 100 = 9300$; $93 \times 2 = 186$; $9300 - 186 = 9114$. After a little practice such solutions may be made silently, the answer only being given. In **9**, first multiply by 10, 30, and 60, and add to the products the multiplicands. Let the pupils perform orally as many of the exercises from **20** to **27** as they can.

83 *Answers:* **1** \$1.92. **2** \$21.675. **3** \$118.625. **4** \$42.98. **5** \$81.15. **6** \$6.75. **7** \$4.59. **8** \$11.66$\frac{2}{3}$ Total, \$289.35$\frac{125}{147}$. **9** 9.5. **10** 1.853$\frac{189}{227}$. **17** \$90666.66$\frac{2}{3}$. **18** \$10125, val. of estate \$5625, wife \$1125, son.

84 *Answers:* **3** 31 A. 139$\frac{151}{251}$ sq. rd. **4** 6$\frac{4}{11}$ lb. **5** 2$\frac{1}{2}$ sq. rd. 1 A. 70 sq. rd. **6** \$31.68. **8** 207$\frac{7}{9}$. **9** 287$\frac{13}{31}$. **10** \$6.53$\frac{1}{3}$. **12** 55¢. \$11.00. \$54

85 *Answers:* **1** 26 poles. **2** 70 posts. **3** 99 posts. **4** 1782 pickets. **5** 2970 stones. **6** 396$\frac{9}{10}$ sheets. **7** \$6.00. **8** \$7.55+. **9** 145 trees. **10** 3$\frac{37}{121}$ sq. rd. **11** 146$\frac{1006}{1089}$ sq. rd. **12** 8$\frac{24}{97}$. **13** 31$\frac{17}{33}$. **14** 69$\frac{11}{21}$. **15** 6$\frac{46}{49}$. **16** 8$\frac{1}{2}$¢. **17** 800 doz. **18** \$25.00. **19** 131.84 pts.

The pupils may be able to perform some of these problems orally. Let plans be drawn for all problems in mensuration. When the steps of a process are not clearly seen, make the reasoning clear by questioning, and use small numbers.

86 *Answers:* **1** 24.99 sq. rd. **2** .025 A. **3** \$16.02. **4** 2800 lb. \$13104. **5** \$240.07½. **16** 844$\frac{16}{21}$ planks. **17** 2150.4 cu. in. 18278.4 cu. in.

87 *Answers:* **1** 4.821 bu. **2** 336.294 bu. 20177.64 lb. **3** $\frac{1}{8}$ 400 ft. 37½ ft. **10** \$9.07½ \$16.335 \$12.10. **11** 46$\frac{2}{3}$ cu. yd. **12** 463 lb. 5$\frac{1}{3}$ oz. 10502$\frac{2}{3}$ lb. **13** $\frac{8}{9}$.

88 *Answers:* **6** $\frac{13}{30}$ $\frac{17}{30}$ 1300 lb. 1700 lb. \$10. **7** \$857$\frac{1}{7}$ \$1714$\frac{2}{7}$ \$3428$\frac{4}{7}$. **8** 5184 lb. **9** 128 flagstones. **10** 1536 bottles. **11** 29.403 lots. **12** 166$\frac{2}{3}$ yd. **13** 45000 oranges. **14** \$62487889.60.

89 *Answers:* **1** 1 mi. 109 rd. 6½ ft. **2** 1$\frac{58}{445}$ sec. **3** 231 rd. 3½ ft. **4** 59 lb. 6 oz. **5** 750 lb. **6** 18600 ft. 3$\frac{23}{44}$ mi. **7** 3 mi. 220 rd. 5$\frac{5}{21}$ ft. **8** 1200 times. **9** $\frac{17}{56}$ week. 8$\frac{5}{14}$ hr. **10** $\frac{185}{372}$. **11** \$2.50 3$\frac{3}{10}$ yd. **12** 1111$\frac{23}{32}$ cu. ft. **14** \$9.53. **15** \$1.50 \$3.75 \$2 \$2.30.

SECTION VIII.

NOTES FOR BOOK NUMBER SIX.

Before taking up the exercises included in this book, the pupils are supposed to have a thorough knowledge of common and decimal fractions and of some of the simpler and more important processes in mensuration and denominate numbers. If these subjects are not well understood, it is advised that a more extended review of Book V. be given than is found in Section I.

It is not unlikely that some parts of mensuration may be thought too difficult; but if the previous work has been well done and each new principle is taught objectively, there will be no difficulty which it will not be well for the pupils to overcome. In the applications

of percentage, also, which include all kinds of problems that are likely to occur in the every-day life of a mechanic or farmer, there will be some difficulties both in the understanding of principles and in the processes involved unless the foundation is carefully laid. The development exercises preceding the practical problems should be carefully and systematically followed if good results are to be expected.

As the work progresses less attention need be given to the mechanical operations with numbers if the previous work has been thoroughly done, while a constantly increasing emphasis should be put upon the solution of problems which require close and careful thinking. Increased attention also should be given to the explanation of processes and the formulation of rules and definitions. With this changed work of the pupils comes a corresponding change in the character of the teaching and drilling. While the use of objects and drawings in making clear a new process or principle is always desirable, there is not so complete dependence upon them as the pupils come to have a fuller power of generalization and reasoning.

Other hints of a general nature will be found in the Note to Teachers, which precedes the first section of the book.

1 *Answers:* **1** 1490. **2** 1392. **3** 1528. **4** 1402. **5** 1588.
6 1448. **7** 1534. **8** 1371. **9** 1537. **10** 1362. **11** 633.
12 769. **13** 607. **14** 664. **15** 624. **16** 612. **17** 492.
18 599. **19** 474. **20** 644. **21** 608. **22** 714. **23** 532.
24 611. **25** 535. **26** 471. **27** 591. **28** 672. **29** 624.
30 673. **31** 700. **32** 471. **33** 757. **34** 548.

Encourage the pupils to add by pairs, thus (**1**): 14, 26, 47, etc. As a convenience it may be well to have the sum of each column placed below the line. For example, the partial and total sums in **1** would appear as follows:

140
135
1490

2 *Answers:* **1** \$49.97. **2** \$50.56. **3** \$61.93. **4** \$55.44.
5 \$55.87. **6** \$56.48. **7** \$54.69. **8** \$46.20. **9** \$51.34.
10 \$49.50. **11** \$46.79. **12** \$35.01. **13** \$41.15. **14** \$47.63.

15 \$34.44. **16** 32.98. **17** \$384.94. **18** \$385.04.
19 \$2506.82. **20** \$2188.85. **21** \$2352.38. **22** \$1710.47.
23 \$1905.60. **24** \$1216.46. **25** \$2395.32. **26** \$258.41.
27 \$2101.83. **28** \$1177.55. **29** \$1376.14. **30** \$2138.41.
31 \$10864.12. **32** \$10664.12. **33** \$848.32 \$340.39 \$788.07 \$389.81 \$508.91. **34** \$62.84 \$355.95 \$888.56 \$381.19 \$574.05. **35** \$315.83 \$611.81 \$0.44 \$676.29 \$339.05. **36** \$323.71 \$600.23 \$318.33 \$49.61 \$269.06. **37** \$427.83 \$664.71 \$331.89 \$525.47 \$36.59. **38** \$84.73 \$665.29 \$650.43 \$30.34 \$722.74. **39** \$1642.54 \$2100.91 \$2191.90 \$1691.54 \$1830.77. **40** \$2490.86 \$1760.52 \$1403.83 \$1291.73 \$1321.86. **41** \$2428.02 \$2116.47 \$2292.39 \$1672.92 \$1895.91. **42** \$2112.19 \$1504.66 \$2291.95 \$1096.63 \$1656.86. **43** \$2435.90 \$2104.89 \$1973.62 \$1146.24 \$1825.92. **44** \$2008.07 \$1440.18 \$2305.51 \$1671.71 \$1862.51. **45** \$776.34 \$412.37 \$6.42 \$289.56 \$13.04 \$249.92 \$500.10. **46** \$72.54 \$520.22 \$12.39 \$623.76 \$294.80 \$701.80 \$613.92. **47** \$131.55 \$529.81 \$22.44 \$553.41 \$185.47 \$8.11 \$688.88. **48** \$45.90 \$165 \$27.86 \$265.10 \$484.55 \$4.33 \$757.41. **49** \$703.80 \$932.59 \$18.81 \$334.20 \$307.84 \$451.88 \$113.82. **50** \$59.01 \$9.59 \$34.83 \$70.35 \$480.27 \$709.91 \$73.96. **51** \$85.65 \$364.81 \$50.30 \$288.31 \$299.08 \$3.78 \$68.53. **52** \$835.35 \$402.78 \$41.25 \$219.21 \$493.31 \$459.99 \$575.06. **53** \$352.18 \$2379.36 \$179.61 \$1707.20 \$1106.63 \$877.39 \$1554.93. **54** \$1128.52 \$1966.99 \$186.03 \$1417.64 \$1093.59 \$627.47 \$2055.03. **55** \$1055.98 \$1446.77 \$198.42 \$2041.40 \$798.79 \$1329.27 \$1441.11. **56** \$1187.53 \$1976.58 \$220.86 \$1487.99 \$613.32 \$1337.38 \$2129.99.

To add in lines as well as in columns is good practice, since additions have to be so made sometimes in business.

3 *Answers:* **1** \$295946306.08. **2** \$20.456+. **9** 715920 1909120 14318400 9545600 71592000 214776000.

10 1232240 1509494 24644800 26185100 26369936 10628070. **11** 1622538 50659242 58651744 516808400 477146360. **12** 37552900 72472220 59401860 690583200 878640320. **13** 2335200 38190250 42082250 34487985 38944325. **14** 569754600 539211785 496692785 5882220190 714319200. **15** $112746\frac{5}{8}$ $72157\frac{21}{25}$ $9019\frac{73}{100}$ $474\frac{113}{190}$ $9698\frac{59}{93}$ $4697\frac{149}{192}$. **16** $300028\frac{1}{5}$ $197386\frac{37}{38}$ $62505\frac{7}{8}$ $76537\frac{79}{98}$ $31782\frac{153}{236}$ $1578\frac{91}{95}$. **17** 125000 80000 $158730\frac{10}{63}$ $22727\frac{3}{11}$ $42016\frac{96}{119}$ $1461\frac{169}{171}$. **18** $572572\frac{2}{7}$ $133600\frac{1}{5}$ $83500\frac{1}{8}$ $318095\frac{5}{7}$ $79682\frac{14}{503}$ $4970\frac{495}{2016}$. **19** $1894779\frac{7}{13}$ $947389\frac{10}{13}$ $761818\frac{56}{97}$ $535481\frac{4}{23}$ $200261\frac{31}{123}$ $151117\frac{63}{163}$. **20** $103621421\frac{14}{29}$ $8232936\frac{52}{73}$ $60707518\frac{98}{99}$ $4732317\frac{121}{127}$ $1397683\frac{29}{43}$ $146729\frac{599}{1024}$.

Problems similar to the following might be given from the table: What was the difference in amount of imports in 1891 and 1892 from England? from France? from Germany? from Brazil? from Mexico? from Cuba? What was the difference in exports in 1891 and 1892 to England? to France? etc. By how much did the exports to England exceed the imports from that country in 1891? in 1892? What can you say of the difference between the exports to France and the imports from that country in 1891? in 1892? etc. Find the total imports in 1891 from England, France, Germany, Brazil, Mexico, and Cuba; etc.

4 *Answers:* **1** $25\frac{25}{51}$. **2** $195\frac{77}{81}$. **3** $1\frac{3}{7}$. **4** $114333\frac{17264}{44555}$. **7** $883.

Let the pupils practice upon exercises in cancellation until they can recognize readily the common factors of dividend and divisor. Answers to **5** and **6** should be given as rapidly as possible without the aid of figures.

5 *Answers:* **52** $60\frac{77}{360}$. **53** $68\frac{143}{240}$. **59** $31\frac{7}{9}$. **60** $49\frac{7}{12}$. **61** $54\frac{37}{90}$. **62** $107\frac{23}{30}$. **63** $106\frac{14}{25}$. **64** 66. **65** $46\frac{52}{63}$. **66** $69\frac{3}{7}$. **67** $60\frac{3}{40}$. **68** $29\frac{1}{4}$.

In **1** to **20** lead the pupils first to find by inspection the least common denominator, and then to add as the reduction is made.

Thus, in **10**, the pupils would say: "The least common denominator is 12; $\frac{9}{12} + \frac{5}{12} = \frac{14}{12}$; $\frac{14}{12} + \frac{3}{12} = \frac{17}{12}$; $\frac{17}{12} + \frac{5}{12} = \frac{22}{12}$; $\frac{22}{12} + \frac{4}{12} = \frac{26}{12} = 2\frac{1}{6}$."

Possibly **21** to **31** may have to be performed by the aid of figures, but the pupils should be given an opportunity to try their solution orally.

6 *Answers:* **1** $111\frac{13}{72}$. **2** $130\frac{2}{3}$. **3** $3\frac{28}{45}$. **11** $\frac{53}{72}$ $2\frac{7}{8}$ $3\frac{1}{24}$ 4 $39\frac{7}{9}$. **12** 800 9 7 16 5. **13** $11\frac{1}{10}$ $1\frac{19}{20}$ $1\frac{7}{9}$ $2\frac{1}{4}$ $1\frac{1}{8}$. **16** 64 $6\frac{11}{16}$ $4\frac{1}{17}$ $46\frac{3}{7}$. **21** 13.9352. **22** 627.3294. **23** 27.0763. **24** 1827.8249.

7 For the sake of practice in pointing off, the solution of **1** to **7** may be written out. All the remaining exercises on this page should be performed orally.

8 All these problems, with the possible exception of **1**, **2**, **11**, and **13**, should be performed orally. Let careful attention be given to the pupils' explanations.

9 Nearly all of these review problems should be performed orally.

10 *Answers:* **1** 100 rd. $34\frac{2}{3}$ rd. **3** $\frac{2000}{4000}$. **4** 5 rd. 3 ft. 10.56 in. **5** \$13.93+. **16** 32670 sq. ft. **17** $5080\frac{1}{2}$ sq. ft. **18** 6076 sq. in. **19** $1\frac{1}{4}$ A. $6\frac{1}{4}$ A. $\frac{1}{8}$ A. **20** $\frac{30}{121}$ A. $\frac{500}{1089}$ A. **21** $\frac{340}{871}\frac{51}{21}$. **22** \$298.07. **23** $6352\frac{1}{2}$ sq. ft. \$1030.75.

11 *Answers:* **1** \$4420.80 318 yd. **4** \$3840 \$2400 \$512.

In places where this method of measurement is common, many exercises similar to **3** and **4** should be given. Descriptions of lots of land found in deeds, advertisements, etc., should be brought into the class and fully explained.

12 *Answers:* **5** 32 yd. breadthwise. **6** 29 yd. 6 in. 27 yd. 8 in. **7** $\frac{3}{4}$ ft. $\frac{1}{4}$ ft. $1\frac{7}{8}$ in. **8** $60\frac{5}{72}$ sq. ft. $16\frac{18}{25}$ courses. **9** 10 sq. ft. 20 sq. ft. **10** 32 sq. ft.

If necessary use strips of paper to illustrate **5** and **6**. Several illustrations similar to that given in **12** will assist the pupils to discover how the average width of a board may be found, and in general how the area of any trapezoid may be found.

13 *Answers:* **7** $26\frac{1}{16}$ sq. ft. **8** $23\frac{243}{1089}$ sq. rd. **9** 160 sq. ft. **10** $11\frac{1}{3}$ sq. yd. **11** $142\frac{2}{3}$ sq. yd. **12** 384 sq. ft. **13** 576 sq. ft. $113\frac{7}{9}$ sq. yd.

The description of plane figures here called for should be more carefully made than that made in answer to the same question in Book V. (page 57). The description should be as nearly as possible in the form of an accurate definition. The pupils should be led first to see that a plane surface is a surface in which a line connecting any two points will lie wholly in the surface, and that a plane figure is a portion of a plane surface bounded by one or more lines. These lines may be straight or curved. Figures bounded by straight lines are rectilinear figures, and figures bounded by curved lines are curvilinear figures. In teaching the definitions of the various figures, first present the figure and call attention by questions to its essential characteristics. For example, after presenting a triangle, the teacher says: "What kind of a figure is this? By how many sides is it bounded? Define triangle." The pupil is led to say: "A triangle is a plane rectilinear figure bounded by three sides." If his wording of the definition is not quite as good as it should be, lead him by questioning to make the desired correction. The following definitions may be developed in the same way:

A polygon is a plane rectilinear figure having three or more sides.

A quadrilateral is a polygon of four sides.

A pentagon is a polygon of five sides. Etc.

A square is a right-angled parallelogram[1] having its sides equal.

A rectangle is a right-angled parallelogram having only its opposite sides equal.

A rhombus is an oblique-angled parallelogram having its sides equal.

[1] This term is supposed to have been taught previously. See Manual, p. 76.

A rhomboid is an oblique-angled parallelogram having only its opposite sides equal.

The walk mentioned in **13** is supposed to be included in the garden. If the blackboards in the room be of the same width, lead the pupils to find first the entire length of all the boards, and then to multiply by the width.

14 *Answers:* **2** \$10.40. **3** \$60 \$44. **4** 43½ sq. ft.

After the pupils have drawn plans illustrating the solution of **3**, lead them to show how they can always find by a short process the length of a walk or trench if made on the inside border of a given rectangular lot, and also its length if made on the outside of the lot. To find the length of a walk if made on the inside border of a rectangular lot, subtract four times the width of the walk from the perimeter of the lot; and to find the length of a walk if made on the outside of such a lot, add four times the width of the walk to the perimeter of the lot.

In the solution of **4**, lead the pupils to connect the corners by dotted lines, and to find by measurement the altitude. That the altitude may be as exact as possible, see that the line measured is exactly perpendicular to the base. The square edge of a card will be a good means of measurement. Let the figures made for **5** be exchanged among members of the class and answers be compared. Let the radius, angles, etc., be designated by letters; thus, *ao*, *aob*, etc.

15 Slow and careful measurements are needed here. Do not take up a new exercise until the pupils thoroughly understand the previous one. The points to be developed in each of the exercises are as follows: **1.** The angles at the centre of a circle are measured by the arcs which subtend them. **2.** A simple way of finding angles of various magnitudes. **3.** To make a protractor for measuring angles. Let this be most carefully made. **4.** Practice in estimating magnitude of angles. **5.** The sum of all the angles made about a given point is equal to 360°. **6.** The opposite angles made by two lines crossing each other are equal. **7.** The sum of the

three angles of a triangle is equal to two right angles. The sum of the angles of a polygon is equal to two right angles taken as many times as the figure has sides less two.

It is not desirable at this time for the pupils to spend much time in formulating the facts. The important point is for them to discover the facts by observation and construction.

16 *Answers:* **6** 258 sq. ft. **7** 143.86+ sq. rd. **8** 46.7694 sq. rd. **9** Exact area = 2781.1584 sq. ft.

Proceed very slowly with the first four exercises, and review frequently so that the meaning of the terms may be fixed in the mind.

The polygon *abcde* is a regular polygon, because it has equal sides and equal angles. The pupils are supposed to have compasses in constructing the figures mentioned in **3**. These two ways of constructing regular polygons should be repeated until they are well understood.

From the questions and illustrations of **4** the pupils can easily discover the process of finding the area of any regular polygon. The pupils should be told that the line *fo* is the *apothem* or *less radius*. The rule for finding the area of a regular polygon should be made by the pupils, and may be as follows: Multiply the perimeter by half the apothem, or multiply half the perimeter by the apothem. The exact length of the apothem of the regular pentagon mentioned in **5** is 8.2584 ft. (.6882 × 12). The measured distance will be, of course, only approximately correct. The ratio of the apothem to the side of a regular polygon is as follows: triangle, .2887; pentagon, .6882; hexagon, .866; octagon, 1.2071; decagon, 1.5388; dodecagon, 1.866.

17 *Answers:* **1** $5628. **2** Neither 71⅓ yd. **3** 12⅔ yd. 49.6 yd. 28⅔ yd. **4** 53⅓ sq. yd. 37⅔ sq. yd. 21½ sq. yd. 21⅓ sq. yd. **5** 611 sq. ft. 25 rolls.

Let the pupils carefully measure the lines whose distances are not indicated. According to the measurement given of the hall, one breadth of the carpet will just fill the narrow hall-way. Four

strips, therefore, will be needed, one strip 20 ft. long and 3 strips each 6 ft. long.

In estimating the amount of paper that will be needed for the walls, only an approximate estimate can be made, as more or less may be needed for matching, etc. A good method is, first to find the number of strips required, then to find how many strips can be cut from a roll. The whole number of strips required divided by the number that can be cut from a roll will give the number of rolls required. Let the pupils perform **6** by this method. The pupils' answers to **6** can be but approximately near the correct answers without definite instructions as to how the wainscot will be placed with reference to chimney, and whether the window in the bay extends below the wainscot. The widest part of the recess of the bay window is supposed to be 10 ft. wide, and the narrowest part 4 ft. wide. Depth of bay, 4 ft.

The given floor plan may be used for other exercises, such as finding the number of sq. ft. of flooring in each room, the cost of painting a border for the floor of each room, etc.

18 *Answers:* **3** 314 sq. ft. **4** 40.13 sq. rd. .2508+ A. **5** 1925 sq. ft. **8** 314.16 sq. ft. 1256.64 sq. ft. **9** 3.183 ft. 15.9154 ft. 2.758 yd. **10** 1.76+ sq. ft. 9.621+ sq. ft. **11** 420.168+ times.

Let the pupils try both ways of finding the area of a circle. The rule for finding the area which the pupils should be led to give, from the questions in **1**, is: "Multiply the circumference by half the radius." And it is by this rule that the pupils should at first be required to find the area of a circle. 3.1416 may be used as the ratio of the circumference to the diameter.

19 *Answers:* **2** 384 sq. in. **3** $7\frac{1}{14}\frac{1}{4}$ sq. ft. **4** 17568 sq. in. **5** \$180 $1292\frac{6}{7}\frac{2}{7}$ gals.

The cube and other right prisms should be presented to the class in teaching these exercises. Right prisms only should be dealt with at first, and when prism is referred to in this book, a right prism is meant. The points of resemblance in all prisms

are: (1) The two bases are parallel. (2) The lateral sides are parallelograms. The definition, therefore, of a prism which may be made by the pupils is: "A solid whose bases are parallel and whose lateral sides are parallelograms."

Only the approximate contents can be given in answer to the questions asked in **8** and **9**, owing to the fact that the exact dimensions of the prisms cannot be found from the cuts. In explaining the process of finding the contents of prisms, lead the pupils at first to use the method given on page 106 of the Manual.

20 *Answers:* **1** 192 cu. in. 364 cu. in. **2** 24⅜ cu. ft. **3** 75 sq. ft. 27.06 cu. ft. **4** 24 sq. ft. 330 cu. ft. **5** 311⅝ sq. ft. 3888 gals. **8** 3 sq. ft. 25.78+ sq. in. 618.72+ cu. in. **9** .319+ cu. in. 5.1051 sq. in.

The work called for in 1 should be done very carefully. If the models are made of cardboard, let the pupils first cut the outline as given, and then cut half through the cardboard where the edges are to be. The figures when folded can be held in place by means of mucilage or paste. Modeling may be continued with profit to the pupils, besides furnishing models for future measurements.

The following facts concerning the cylinder should be taught objectively and definitions developed:

A cylinder (right cylinder) is a solid bounded by a curved surface and by two equal parallel circles. The curved surface is called the convex surface, and the other two sides are called the bases.

To find the convex surface of a cylinder, multiply the circumference of the base by the height.

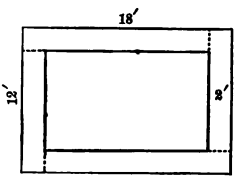
To find the volume of a cylinder, multiply the surface of one of the bases by the height.

21 *Answers:* **1** $40.72+. **2** $113.10−. **3** 706.86 cu. in. **4** 1800 cu. in. 5 in. **5** 8 in. **6** 10 ft. **7** 2 ft. **8** 4½ ft. **9** 16 ft. 8 in. **10** 3911.38+ in. **11** 8⅔ sq. yd. **12** $17.79−. **13** 8¼ cu. ft. **14** 3630 cu. ft. **15** 640 sq. ft. $9.60 $5.12. **16** 17½ yd.

After solving several simple problems similar to **5**, let the rule be given for finding one dimension of a solid when the cubic contents and two dimensions are given. Whenever any point is not clear, use models and small numbers. Let the pupils illustrate all processes by drawings when it can be done.

22 *Answers:* **1** 56 sq. yd. $62\frac{2}{9}$ sq. yd. 24 yd. **2** 40 ft. 320 sq. ft. **3** 486 sq. ft. 54 sq. yd. **4** $35\frac{1}{3}$ sq. yd. **5** 8 rolls. **8** 104 sq. ft. $12\frac{4}{9}$ sq. yds. **9** 272 sq. ft. \$13.60. **10** 18 yd. **11** $26\frac{2}{3}$ yd. widthwise of the room.

For method of finding the approximate number of rolls of paper required to paper the walls of a room, see Manual, page 119. Let **5** be performed by that method. Great care should be taken in the solution of all these problems to designate what each concrete number stands for. Another good way is to indicate the process in one place and perform the indicated operations in another place. For example, in **8**.



$[(16 \times 2) + (10 \times 2)] \times 2 =$ sq. ft. in border.

$\frac{(18 \times 12) - 104}{9} =$ sq. yd. in floor unpainted.

$$\begin{array}{r} 32 \\ 20 \\ \hline 52 \\ 2 \\ \hline 104 \end{array}$$ sq. ft.

$$\begin{array}{r} 216 \\ 104 \\ \hline 9\overline{)112} \end{array}$$ $12\frac{4}{9}$ sq. yd.

or,

$(18 \times 12) - [(18 - 4) \times (12 - 4)] =$ sq. ft. in border.

$\frac{14 \times 8}{9} =$ sq. yd. in floor unpainted.

$$\begin{array}{r} 216 \\ 112 \\ \hline 104 \end{array}$$ sq. ft.

$$\begin{array}{r} 14 \\ 8 \\ \hline 9\overline{)112} \end{array}$$ $12\frac{4}{9}$ sq. yd.

Either of these solutions or any other correct solution should be accepted, and may be followed by an oral explanation.

If the pupils are not considered ready for such work as that noted above, they should be led to write out the solution, so as to indicate that they understand every step. The following is a sample of what should be expected:

18 — 18 ft. − 4 ft. = 14 ft., length of unpainted part.
12 — 12 ft. − 4 ft. = 8 ft., width " " "
216 sq. ft. in entire floor. — 14
112 " " " unpainted part. — 8
104 " " " border. — 112 sq. ft. in unpainted part.
9 sq. ft.)112 sq. ft.
$12\frac{4}{9}$ = no. sq. yd. in unpainted part.

23 *Answers:* **1** $20\frac{8}{9}$ bd. ft. **2** $19\frac{1}{16}$ bd. ft. **3** 96 ft. **4** 2652 ft. $42.43. **5** $1421\frac{1}{8}$ bd. ft. $21.32. **6** $12\frac{4}{27}$ loads. **7** $1564\frac{4}{9}$ loads. **8** $376.32. **9** 2.9+ pailfuls. **10** $172\frac{4}{5}$ 173.57+ bu. 144 bu. 5 T. 8 cwt. **11** 4700.16 gal. **12** 34.27+ rd. 93.46+ sq. rd.

24 *Answers:* **1** $23\frac{1}{3}$ in. **2** 5 ft. 6.84 in. **3** 36 sq. ft. **4** $70\frac{1}{4}$ sq. ft. **5** $56.10 $3234.33.

Dictation exercises similar to **6** will be found interesting and profitable to the pupils. Original problems of the same kind should be made and brought into the class.

25–27 If the measures have been properly used in previous grades, they need not be used in the solution of these problems, most of which should be performed orally. Let the pupils perform the problems in the shortest and most direct way. Correct and lead by questioning rather than by giving outright the shorter method. For example, if the pupils attempt to perform **24**, page 26, in a long or roundabout way, ask such questions as: "What short way of getting the cost of 25 bu. at 75¢ a bushel? (Probably two ways will be given: one, $\frac{1}{4}$ of 100 times 75¢, and the other, 25 times $\frac{3}{4}$ of a dollar.) How much is it? 3 pk. 6 qt. is how many quarts less than a bushel? What part of a bushel is it? 3 pk. 6 qt., then, will cost how much less than 75¢?" After answering these questions the pupils can easily perform the

problem, and will say: "1 bu. costs $\frac{3}{4}$ of a dollar; 25 bu. will cost 25 times $\frac{3}{4}$ of a dollar or $\frac{75}{4}$, equal to \$18.75. 3 pk. 6 qt. is 2 qt. or $\frac{1}{16}$ of a bushel less than a bushel, and will therefore cost $\frac{1}{16}$ of 75¢ less than 75¢. $\frac{1}{16}$ of 75¢ is 4 cents and a fraction, subtracted from 75¢ equals 71¢. This sum added to \$18.75 is \$19.46, the cost of 25 bu. 3 pk. 6 qt. of wheat at 75¢ a bushel."

28–30 If possible, bring to the class the various coins referred to in these exercises, and talk about their absolute and relative values. Some of the solutions may need the aid of figures, but most of them can be performed orally. Information in regard to coins not contained here will be found in the Appendix of Book VIII.; also on page 89, Book IV., and page 69, Book V., and notes thereon in the Manual.

31 *Answers:* **10** 12 lb. 4 oz. **11** 12 cwt. 20 lb. **12** 6 T. 10 cwt. 30 lb. **13** 16 T. 5 cwt. 50 lb. **14** 1 lb. 7 pwt. **15** 4 lb. 2 oz. 2 pwt. **16** 2ʒ. **17** 9*f*ʒ 4*f*ʒ 10℔. **18** 8 cwt. 91 lb. 13 oz. 9 T. 18 cwt. 8 lb. 12 oz. **19** 10ʒ 2ȝ 2ϑ 6*f*ʒ 16℔. **20** 4 T. 18 cwt. **21** 3 T. 11 cwt. 82 lbs. **22** 9 oz. 4 pwt. **23** 2ʒ 1ȝ 2ϑ 17 gr. **24** 4ȝ 2ϑ 5 gr. 7 lb. 7 oz. 3 dr. 1 sc. **25** 1 T. 1 cwt. 60 lb. 3 T. 12 cwt. 20 lb.

32 *Answers:* **1** 4 lb. 5ʒ. **2** 9*f*ʒ 4*f*ȝ. **3** 27 T. 3 cwt. 60 lb. **4** 8 lb. 1 oz. 15 pwt. **5** 9 T. 17 cwt. 2 lb. 8 oz. 112 T. 12 cwt. 80 lb. **6** 38 lb. 1ϑ 10 gr. 148 cong. 4 O. **7** 19 cwt. 55 lb. $8\frac{8}{9}$ oz. **8** 3 oz. 10 pwt. $3\frac{1}{3}$ gr. **9** 3ȝ 2ϑ 5 gr. **10** 2ȝ 16℔. **11** 9 cwt. 89 lb. $5\frac{1}{3}$ oz. 5 oz. 2 pwt. 18 gr. **12** 537.5 lbs. \$8.06. **13** 3 oz. 15 pwt. 12 gr. **14** $\frac{21}{80}$ T. **15** $\frac{17}{180}$ lb. **16** \$240. **17** \$2.05. **18** 1*f*ʒ 2*f*ȝ. **19** 7 lbs. 1 oz. 16 pwt. 16 gr. 71 lb. 6 oz. 6 pwt. 16 gr. **20** \$89.08. **21** 200 pills. 360 pills. **22** 28, and 37 pwt. left.

33 *Answers:* **2** *J* 640 rd. *K* 1920 rd. *L* 960 rd. *M* 1280 rd. *N* 320 rd. *O* 2880 rd. *P* 2560 rd. *Q* 2240 rd. **3** *J* 87120 sq. ft. *K* 261360 sq. ft. *L* 130680 sq. ft. *M* 174240 sq. ft. *N* 43560 sq. ft. *O* 392040 sq. ft. *P* 348480 sq. ft. *Q* 304920 sq. ft. **4** *J* 48 qt. *K* 60 qt. *L* 40 qt. *M* 52 qt. *N* 24 qt.

O 32 qt. *P* 44 qt. *Q* 56 qt. **5** *J* 64 qt. *K* 192 qt. *L* 96 qt. *M* 128 qt. *N* 32 qt. *O* 288 qt. *P* 256 qt. *Q* 224 qt. **6** *J* 16*s*. *K* 12*s*. *L* 5*s*. *M* 15*s*. *N* 1*s*. *O* 2.5*s*. *P* 7.5*s*. *Q* 12.16*s*. **7** *J* 1320 gr. *K* 840 gr. *L* 1860 gr. *M* 3264 gr. *N* 2700 gr. *O* 2320 gr. *P* 3640 gr. *Q* 1696 gr. **8** *J* $\frac{31}{728}$ A. *K* $\frac{397}{4356}$ A. *L* $\frac{34}{363}$ A. *M* $\frac{224}{1815}$ A. *N* $\frac{601}{4840}$ A. *O* $\frac{3029}{21780}$ A. *P* $\frac{1987}{21780}$ A. *Q* $\frac{292}{1815}$ A. **9** *J* $\frac{93}{100}$ T. *K* $1\frac{187}{200}$ T. *L* $2\frac{2}{50}$ T. *M* $2\frac{86}{125}$ T. *N* $2\frac{509}{2000}$ T. *O* $3\frac{38}{1000}$ T. *P* $1\frac{987}{1000}$ T. *Q* $3\frac{63}{125}$ T. **10** *J* 880 yd. *K* 1320 yd. *L* 1100 yd. *M* $733\frac{1}{3}$ yd. *N* $1026\frac{2}{3}$ yd. *O* $938\frac{2}{3}$ yd. *P* 1496 yd. *Q* $1114\frac{2}{3}$ yd. **11** *J* $232\frac{1}{2}$ cd. *K* $496\frac{1}{4}$ cd. *L* 5010 cd. *M* 672 cd. *N* $563\frac{5}{8}$ cd. *O* $757\frac{1}{4}$ cd. *P* $496\frac{3}{4}$ cd. *Q* 876 cd. **12** *J* 960 gi. *K* 1920 gi. *L* 2880 gi. *M* 1168 gi. *N* 3344 gi. *O* 5792 gi. *P* 5088 gi. *Q* 4320 gi. **13** *J* 14520 ft. *K* 9240 ft. *L* 20460 ft. *M* 35904 ft. *N* 29700 ft. *O* 25520 ft. *P* 40040 ft. *Q* 18656 ft. **14** *J* 25.6 qt. *K* 19.2 qt. *L* 8 qt. *M* 24 qt. *N* 1.60 qt. *O* 4 qt. *P* 12 qt. *Q* 19.456 qt. **15** *J* 320 A. *K* 480 A. *L* 400 A. *M* 266 A. 106 sq. rd. $181\frac{1}{2}$ sq. ft. *N* 373 A. 53 sq. rd. $272\frac{1}{4}$ sq. ft. *O* 341 A. 53 sq. rd. $272\frac{1}{4}$ sq. ft. *P* 544 A. *Q* 405 A. 53 sq. rd. $272\frac{1}{4}$ sq. ft. **16** *J* 1000 lb. *K* 1500 lb. *L* 1250 lb. *M* 833 lb. $5\frac{1}{3}$ oz. *N* 1166 lb. $10\frac{2}{3}$ oz. *O* 1066 lb. $10\frac{2}{3}$ oz. *P* 1700 lb. *Q* 1266 lb. $10\frac{2}{3}$ oz. **17** *J* 80 lb. *K* 60 lb. *L* 25 lb. *M* 75 lb. *N* 5 lb. *O* 12.5 lb. *P* 37.5 lb. *Q* 60.8 lb. **18** *J* 256 rd. *K* 192 rd. *L* 80 rd. *M* 240 rd. *N* 16 rd. *O* 40 rd. *P* 120 rd. *Q* 194 rd. 9 ft. 2.88 in. **19** *J* 11 qt. *K* 7 qt. *L* 15 qt. 1 pt. *M* 27 qt. $1\frac{3}{5}$ gi. *N* 22 qt. 1 pt. *O* 19 qt. $2\frac{3}{8}$ gills. *P* 30 qt. $2\frac{3}{8}$ gi. *Q* 14 qt. $1\frac{1}{15}$ gi. **20** *J* 54 oz. *K* 75 oz. *L* 52 oz. 16 pwt. *M* 73 oz. 10 pwt. *N* 101 oz. 2 pwt. 17.28 gr. *O* 76 oz. 12 pwt. 15.36 gr. *P* 121 oz. 1 pwt. 14.4 gr. *Q* 216 oz. 1 pwt. 4.8 gr. **21** *J* 13 cu. ft. 864 cu. in. *K* 20 cu. ft. 432 cu. in. *L* 16 cu. ft. 1512 cu. in. *M* 11 cu. ft. 432 cu. in. *N* 15 cu. ft. 1296 cu. in. *O* 14 cu. ft. $691\frac{1}{3}$ cu. in. *P* 22 cu. ft. $1641\frac{3}{8}$ cu. in. *Q* 17 cu. ft. $172\frac{4}{5}$ cu. in. **22** *J* 52 rd. *K* 75 rd. *L* 42 rd. *M* 91 rd. *N* 52 rd. *O* 60 rd. *P* 84 rd. *Q* 82 rd. **23** *J* 133 T. *K* 258 T. 10 cwt. *L* 236 T. *M* 346 T. 16 cwt. *N* 271 T. 9 cwt. *O* 354 T. 1800 cwt. *P* 271 T. 1400 cwt. *Q* 418 T. 800 cwt. **24** *J* 162 rd. 4 yd. $4\frac{1}{2}$ in.

K 241 rd. 4 yd. 4½ in. *L* 203 rd. 4 yd. 2 ft. 5¼ in. *M* 140 rd. 2 ft. 2⅜ in. *N* 192 rd. 1 yd. 1 ft. 9¾ in. *O* 175 rd. 2 yd. 2 ft. 3 in. *P* 279 rd. 3 yd. 7½ in. *Q* 206 rd. 2 yd. 2¼ in. **25** *J* 13 A. 100 sq. rd. *K* 30 A. 130 sq. rd. *L* 25 A. 80 sq. rd. *M* 37 A. 96 sq. rd. *N* 29 A. 29 sq. rd. *O* 46 A. 138 sq. rd. *P* 32 A. 134 sq. rd. *Q* 50 A. 124 sq. rd. **26** *J* 1 bu. 3 pk. 5 qt. 1.2 pt. *K* 2 bu. 5 qt. 0.4 pt. *L* 1 bu. 1 pk. 3 qt. 0.4 pt. *M* 2 bu. 1 pk. 1 qt. *N* 2 bu. 5 qt. 0.048 pt. *O* 1 bu. 2 pk. 7 qt. 0.176 pt. *P* 2 bu. 3 pk. 4 qt. 1.44 pt. *Q* 5 bu. 3 qt. 0.992 pt. **27** *J* 16 cu. ft. 432 cu. in. *K* 22 cu. ft. *L* 20 cu. ft. 1296 cu. in. *M* 18 cu. ft. 86.4 cu. in. *N* 21 cu. ft. 648 cu. in. *O* 19 cu. ft. 403.2 cu. in. *P* 1 cu. yd. 3 cu. ft. 3240 cu. in. *Q* 20 cu. ft. 1094.4 cu. in. **28** *A* 4 rd. *B* 3 rd. *C* 20 rd. *D* 120 rd. *E* 2110 rd. *F* ¼ rd., or 4 ft. 1½ in. *G* 0.2 rd., or 3 ft. 3.6 in. *H* 1 rd. *I* 1.75 rd., or 1 rd. 4 yd. 4½ in. **29** *A* 5 gal. 1 qt. *B* 12 gal. 2 qt. *C* 52 gal. *D* 150 gal. *E* 2950 gal. *F* 2 qt. 3 gi. *G* 2 qt. 1.2 gi. *H* 3 qt. 1 gi. *I* 5 gals. 1 pt. 0.8 gi. **30** *J* 40 sq. rd. *K* 60 sq. rd. *L* 68 sq. rd. *M* 11⅛ sq. rd. *N* 47⅛ rd. *O* 33⅛ sq. rd. *P* 63 sq. rd. *Q* 33⅛ sq. rd. **31** *J* 2 bu. *K* 0 pk. *L* 2 bu. *M* 6 bu. 2 pk. *N* 5 bu. 2 pk. *O* 5 bu. *P* 7 bu. 1 pk. *Q* 3 bu. **32** *J* 123.5 sq. rd. *K* 89.75 sq. rd. *L* 35.6 sq. rd. *M* 113.875 sq. rd. *N* 428 sq. rd. *O* 13.614 sq. rd. *P* 49.91 sq. rd. *Q* 79.275 sq. rd. **33** *J* $7\frac{1}{4}s$. *K* $13\frac{1}{4}s$. *L* $8\frac{5}{8}s$. *M* $1\frac{8}{11}s$. *N* $6\frac{1}{24}s$. *O* $5\frac{5}{6}s$. *P* $9\frac{5}{12}s$. *Q* $9\frac{2}{15}s$. **34** *J* $13\frac{53}{88}$ qt. *K* $17\frac{5}{11}$ qt. *L* $11\frac{7}{11}$ qt. *M* $15\frac{7}{55}$ qt. *N* $6\frac{54}{88}$ qt. *O* $9\frac{17}{55}$ qt. *P* $12\frac{2}{5}$ qt. *Q* $16\frac{18}{55}$ qt. **35** *J* $180. *K* $375. *L* $140.80. *M* $477.75. *N* $387.69−. *O* $332.07+. *P* $736.57. *Q* $1224.34. **36** *J* $17.88−. *K* $11.38−. *L* $25.19−. *M* $44.20. *N* $36.56+. *O* $31.42−. *P* $49.29+. *Q* $22.97−. **37** *J* $30.61+. *K* $45.92−. *L* $38.27−. *M* $25.51−. *N* $35.72−. *O* $32.65+. *P* $52.04+. *Q* $38.78−.

34 *Answers:* **1** Apr., June, Sept., Nov.; Jan., Mar., May, July, Aug., Oct., Dec.; Feb. **2** 365 52 wk. 1 da. 366 da. **4** $546 or $547.75. **5** 4 mo. 7 da. 6 mo. 10 da. 2 mo. 6 da. **6** 7 yr. 1 mo. 29 da. 22 yr. 2 mo. 29 da. **7** 400 da. 473 da. 298 da.

8 $342.08. **10** 67 yr. 9 mo. 22 da. **11** April 15, 1865. **12** Goethe: 82 yr. 6 mo. 23 da. Longfellow, 75 yr. 0 mo. 25 da. Luther, 62 yr. 3 mo. 8 da. Franklin, 84 yr. 3 mo. 0 da. Shakespeare, 52 yr. Napoleon I., 51 yr. 8 mo. 20 da. Burns, 37 yr. 5 mo. 26 da. Milton, 65 yr. 10 mo. 30 da.

The number of days in each month of the year should be repeated until it can be told as soon as the name of the month is given.

A good method of finding the months and days from one date to another is illustrated by the following solution of **5**: From Aug. 13 to Dec. 13 is 4 mo.; from Dec. 13 to Dec. 20 is 7 da. *Answer:* 4 mo. 7 da. From June 30 to Aug. 30 is 2 mo.; from Aug. 30 to Sept. 5 = 1 day in August + 5 days in Sept. = 6 da. *Answer:* 2 mo. 6 da. And in **12**: From 1749 to 1831 is 82 yr.; from Aug. 28 to Feb. 28 is 6 mo.; from Feb. 28 to Mar. 22 is 22 da. *Answer:* 82 yr. 6 mo. 22 da.

A common custom in banks is to regard 30 days for every month in estimating time. For example, in finding the time between June 30 and Sept. 5 the process would be: From June 30 to Sept. 30 = 3 mo. − 25 da. (the time from Sept. 5 to Sept. 30) = 2 mo. 5 da.

In performing the latter part of **4**, let the pupils find the amount saved if the year begins on a week-day, and, again, if the year begins on Sunday; also if the year is a leap year.

To find the exact number of days from one date to another (**7**): From Sept. 16, 1891, to Sept. 16, 1892, is 366 da.; 14 more days in Sept. + 20 in Oct. = 34 da., which, added to 366, equals 400 da.

35 *Answers:* **4** 378′ 754′. **5** 1820″ 2558″. **6** 10848″ 14424″. **7** 5° 17° $7\frac{1}{2}$°. **8** .3 of a degree .6 of a degree .275 of a degree $\frac{22}{36}$°. **9** 22° 12′. **10** New York, 74° W. Baltimore, 76° 35′ W. Chicago, 87° 35′ W. San Francisco, 122° 25′ W. **11** 16° 33′ 30″. $861\frac{1}{30}$+ mi. **13** 21600 geog. mi. 24840 statute mi.

Lead the pupils to use the globe and outline map in estimating distances. Show by means of the globe the reason for the difference in length of the degree of longitude in various latitudes. Some of the distances are given in the Appendix of Book VIII.

36 *Answers:* **1** 192 mi. 12.5 hr. 5 da. 5 hr. **2** 21 da. $1\frac{5}{7}$ hr. **3** 312 lb. 8 oz. **4** 3 T. 750 lb. **5** 537.605 cu. in. 67.201 cu. in. **6** 19.286 bu. **7** 247.76 bu. **8** 198.2 bu. **9** 97.71+ bu. **10** 64.28+ qt. **11** (*a*) 1.75 M. (*b*) 3.50 M. (*c*) 1.29¢ (*d*) 1.29¢ (*e*) 25.8¢ (*f*) \$4.902.

For all practical purposes, in estimating the capacity of bins, the bushel, stricken measure, may be regarded as containing $1\frac{1}{4}$ cu. ft. The answers above given are found by exact measure. Show that a bin or box will hold $\frac{4}{5}$ as many bushels by heaped measure as it will by stricken measure, on the supposition that the heaped measure will hold $\frac{1}{4}$ more than the stricken measure.

37 *Answers:* **1** $3\frac{3}{49}$¢. **2** \$1104. **8** \$65.75 gain. **9** \$12. **10** \$120. **11** 7560 bricks 648 bricks less. **12** 158 rd. 6.6 ft. **13** $7\frac{1}{17}$ loads. **14** $733\frac{1}{3}$ loads.

The solution of problems on a line sometimes shortens the process, as shown in the following solution of **1**:

$$\frac{\$6.00}{196} \times (\tfrac{4}{5} \text{ of } 1\tfrac{1}{2}) = \frac{\overset{.60}{\$6.\cancel{00}} \times \cancel{4} \times 3}{\underset{49}{\cancel{196}} \times \cancel{5} \times \cancel{2}} = 3\tfrac{3}{49}¢.$$

The tons mentioned in **8** are short tons. Performed on a line, the solution is:

$$\$5.25 \times \frac{128.3 \times \overset{100}{\cancel{2000}}}{\underset{112}{\cancel{2240}}} - (\$5.20 \times 128.3).$$

Bricks vary greatly in size, the dimensions here used for **11** being $8\frac{1}{4}'' \times 4'' \times 2\frac{1}{4}''$. Let the pupils see how the bricks must be placed in both instances to make a pile of the given dimensions.

The common bricks are placed on edge, and the Milwaukee bricks are laid flat.

38 *Answers:* **1** 4 T. 1800 lb. **2** $70\frac{8}{15}$ bbl. $183\frac{8}{15}$ bbl. $89\frac{14}{15}$ bbl. **3** 17.5 T. **4** 25 T. 1440 lb. **5** 11600 lb. $59\frac{9}{15}$ bbl. **6** \$36.80. **7** $\$2.73\frac{7}{75}$. **8** $3824\frac{1}{8}$ mi. **9** 9 A. 87 sq. rd. 24 sq. yd. $6\frac{1}{4}$ sq. ft. **10** 160 rd. 6 mi. 230 rd. **11** 36 mi. $226\frac{3}{8}$ mi. $14\frac{1}{8}$ mi. **12** $95\frac{5}{8}¢$. **13** 556 bu. \$444.80 \$300.24. **14** $114\frac{3}{7}$ lb. 2 bu. $1\frac{1}{4}\frac{5}{8}$ qt. **15** 34 cu. ft. $1483\frac{5}{17}$ cu. in.

It is assumed in **7** that the flour bought is whole wheat flour.

39 In addition to the helps here given, all the common measures of the metric system should be used for the exercises of this section. Much time should be given to actual measurements before the exercises are taken.

If a meter-stick is not provided, each pupil can easily make one by cutting a stick exactly ten times as long as the decimeter shown in the cut. The stick can be divided by cross-lines into decimeters and centimeters.

40 *Answers:* **7** 8.64^{Km} 86.4^{Hm} 864^{Dm}. **9** 8570.704^{m}. **10** 9030.043^{m}. **11** 312.5 times. **12** 2280^{m}. **13** \$5.74. **14** 70601 poles.

Similar exercises may be given for class drill if needed. Show two ways of reading metric numbers; thus, 69.83^{m} may be read sixty-nine and eighty-three hundredths meters, or, sixty-nine meters and eighty-three centimeters. Let the pupils practice in reading numbers in both ways.

41 *Answers:* **2** $10000^{\text{q cm}}$. **3** $1000000^{\text{q m}}$. **5** $.0001^{\text{q Km}}$ $.000001^{\text{q Km}}$ $.001^{\text{q Km}}$. **6** $1000^{\text{q m}}$ $100^{\text{q m}}$ $100^{\text{q m}}$. **7** $9.603976^{\text{q Km}}$. **8** $8.697544^{\text{q m}}$. **11** $6849602.1^{\text{q m}}$. **12** $5005000.8^{\text{q m}}$. **13** $251.125^{\text{q m}}$. **14** $200000^{\text{q m}}$. **15** 12500 bricks. **16** 5^{a} $.5^{\text{a}}$ 500^{a} $.05^{\text{a}}$. **17** 200^{a} 4^{a} $.08^{\text{a}}$ 6000^{a}.

The sign for square is sometimes written sq. instead of q. Call attention to the fact that, as 100 units of one denomination make a unit of a denomination next larger, two places of figures are allowed for each denomination.

42 *Answers:* **1** \$467.50. **2** \$500. **3** \$320 gain. **4** 2000 ^{qm} 20 ^{a}. **6** 0.001 $^{cu\,m}$ 0.000001 $^{cu\,m}$ 0.000000001 $^{cu\,m}$. **7** One thousand times as large. **8** One million times as large. **9** 8048000 $^{cu\,cm}$. **10** 0.0085 $^{cu\,m}$. **11** 357 $^{cu\,m}$ 142.8 loads. **12** 1.4476+ $^{cu\,m}$. **13** 120 st. **14** 58.5 st. **15** \$294.84.

43 *Answers:* **3** 800 ^{l} 60 ^{l} 0.8 ^{l} 4 ^{l} 900 ^{l}. **4** 8 ^{Kl} 0.78 ^{Kl} 4 ^{Kl}. **5** 450 ^{dl} 0.8 ^{dl} 50 ^{dl} 40000 ^{dl}. **6** 0.02 ^{l}. **7** 6000 ^{l} 60 ^{Hl}.

The liter measure is one decimeter long, wide, and deep. If the school is not supplied with a liter, one can be made of wood or tin. This measure, filled with water at a temperature a little above freezing, weighs a thousand grams, or one kilogram. These facts should be given to show the relation that the measures of volume, capacity, and weight have with one another, all depending upon the meter, which is supposed to be the ten-millionth part of the distance on a meridian from the equator to the pole.

44 *Answers:* **1** 400 ^{g} 8000 ^{g} 9000 ^{g} 80 ^{g} 60 ^{g}. **2** 834.971 ^{g}. **3** 609.801 ^{g}. **5** 86.97 ^{g} 84 ^{g}. **6** 86 ^{g}. 1000 ^{g}. **8** 1 ^{Kg}. **9** 12000 ^{Kg}. **10** 24000 ^{Kg}.

The last four exercises on the page are important only as they serve to give the pupils an idea of the value of the common metric measures in measures familiar to them. It may not be necessary for all these exercises to be given to accomplish the purpose.

45 *Answers:* **2** 2$\frac{2}{3}$ hr. **3** \$200 \$21.50. **4** 1341$\frac{2}{3}$ ^{cm}. **5** 13.88 ^{Km}. **7** 3675360 ^{Kg}. **8** 2323.2 ^{Hl}. **9** 72000 ^{l}. **10** 25.8 st. **11** 5 ^{m}. **12** .045024 $^{q\,Km}$ \$36019.20. **13** 7166$\frac{2}{3}$ $^{q\,m}$ \$48. **14** 399700 $^{q\,m}$. **15** 900 ^{m}.

46 *Answers:* **1** \$20.34. **2** 27000 ^{l}. **3** 3.6 $^{cu\,m}$ 3600 ^{l}. **4** 51.072 $^{cu\,m}$ 31.92 loads. **5** 10 hr. **6** 1 ^{Kg}. **7** 4297.674 $^{q\,m}$ 42.97674 ^{a}. **8** 9 st. **9** 4.8 ^{m}. **10** 3 ^{T} 447 ^{Kg}. **11** 12.5 ^{Kg}. **12** 7112+ half-dollars. **13** 45208.8 ^{g}. **14** \$7188. **15** 30.35 ^{a}.

One important end should be gained in work upon the metric system, and that is, to give pupils a clear idea of the simplicity of the system in comparison with our system in common use, and

the great saving of time which its adoption would occasion. If it is desired that pupils shall be acquainted with the denominations of the system well enough to perform problems at any time, frequent reviews will be necessary.

47–49 Follow carefully the order of exercises here given, and let the pupils practice upon them until they can perform them readily. Their understanding of subsequent work will depend largely upon the thoroughness with which they take this preliminary work.

50 *Answers:* **1** 0.696 2.7130$\frac{2}{3}$ 75.1152. **2** 296.1 447.43$\frac{1}{8}$ 0.033026. **3** 923.68 45.08 44.54$\frac{1}{8}$. **4** \$35.8234 17.4147$\frac{1}{4}$ 2550.62$\frac{1}{2}$. **5** \$44.889$\frac{1}{4}$ 1134.3392 886.033 mi. **6** 2000.882 \$154.26$\frac{1}{4}$ 142.47$\frac{1}{2}$. **7** 0.0039501 0.21375 47.66625 A. **8** 9.945 0.5899 29.41. **9** 0.01$\frac{1}{18}$ 0.437 0.684988. **10** 2.80$\frac{1}{8}$ 0.002$\frac{7}{8}$. 0.0001369$\frac{3}{8}$. **19** 1.852 0.03538 0.002$\frac{8}{7}$. **20** \$64.8284 1.3008$\frac{1}{2}$ 0.07$\frac{9}{17}$. **21** 23.465 0.39$\dot{6}$2425 0.06$\frac{5}{12}$. **22** 0.063615 0.002$\frac{1}{33}$ 35.6325. **23** 22633.65 1980.68 55.8723$\frac{3}{4}$. **24** \$0.17226 \$16.25525 0.004$\frac{8}{9}$. **25** 0.003$\frac{1}{3}$ 217.875 0.0033432$\frac{7}{8}$. **26** 2102.118 hr. 0.012$\frac{6}{25}$ 15.772328. **27** 43.6720$\frac{3}{8}$ 40559.44$\frac{3}{8}$. 0.247$\frac{1}{8}$. **28** 0.05$\frac{1}{8}$ 0.000032 19630.

51 Nearly all of these problems should be performed orally, but for the sake of learning a good form of written analysis, the pupils might write out the solution of some of them. The following form of solution is suggested:

14 Given: Cost of farm.
Required: The selling price.

\$3550.00 cost of farm.
.33$\frac{1}{3}$
\$1183.33$\frac{1}{3}$ gain.
3550.00
\$4733.33$\frac{1}{3}$ selling price.

52 *Answers:* **1** \$6375. **2** \$.0807. \$.1521. **3** \$1456. **4** \$5098. **5** 383520 lb. 940 bu. **6** 97$\frac{1}{7}$ bu. **7** \$630. **8** \$84.81. **9** 3031.2 bbl. **10** 38250. **11** \$21$\frac{7}{8}$ loss. **12** \$359.13. **13** \$49.93.

Lead the pupils in such problems as **13** to shorten the process as much as possible. In this problem they would say: "There are 332 gallons in all. If 8% of the molasses was lost and 45% of it was sold, there would be left 47% of it. 47% of 332 is 156.04 gallons. 32 cents, the price of a gallon, multiplied by the number of gallons is $49.93."

53 *Answers:* **1** 7 cwt. 9.5 lb. **2** $2523.17. **3** 1.64$\frac{2}{3}$ yd. **4** $112.50. **5** $1827.50. **6** $117. **7** $2100000. **8** 787828.29$\frac{1}{7}$ tons. **9** 640485.449 tons. **10** 1099962. **11** 1515111.54. **12** $1450.80. **13** 6 lb. **14** $43.80.

When these problems have been solved and solutions in good form written out, the pupils might practice in writing the solution on a line, using hundredths for per cent, as in **12**, as follows:

$$\frac{\$3 \times 186 \times 40}{4 \times 100} + \frac{\$4 \times 186 \times 60}{5 \times 100} = \$1450.80.$$

(93 over 186; 2 over 40)

54–55 A new principle is involved in these exercises. In previous exercises the base and rate were given to find the percentage. In these which follow, the base and percentage are given to find the rate. These terms need not be given to the pupils at present. They should be led to recognize the conditions after sufficient practice. Whenever the rate per cent that one number is of another is called for, let the pupils first get the common fractional part and then reduce the fraction to hundredths, or per cent. Great care should be taken to lead the pupils to recognize the base, or the number of which another number is a part. The form of question should be variously given, so that the pupils will not work mechanically. Sometimes the base is the first number given in the question, as, What part of 8 is 4? and sometimes the percentage is the first number given, as, 4 is what part of 8? Sufficient practice in both forms should be given, so as to enable the pupils to recognize the base wherever it is placed in the question.

56 *Answers:* **1** 12.5% $8.69\frac{1}{21}$% $197\frac{1}{253}$% $166\frac{2}{801}$% $25\frac{5}{78}$%. **2** 67.5% $\frac{12}{11}$% $46\frac{6}{14}$% $\frac{25}{37}$% $347\frac{13}{21}$%. **3** $1\frac{68}{337}$% $\frac{35}{65}$% $14\frac{2}{37}$% $\frac{125}{305}$% $114\frac{22}{137}$%. **4** $40\frac{7}{12}$% $2\frac{31}{105}$% $266\frac{2}{253}$% $29\frac{595}{801}$% $4\frac{21}{350}$%. **5** $24\frac{28}{247}$% $13\frac{1}{7}$% $18\frac{1}{2}$% $12\frac{5}{25}$% $39\frac{1}{18}$. **6** $1\frac{28}{337}$% $90\frac{6}{138}$% $30\frac{10}{37}$% $59\frac{268}{305}$% $33\frac{8}{127}$. **7** $30\frac{320}{487}$% $389\frac{1}{241}$% $74\frac{198}{673}$% $554\frac{18}{225}\frac{38}{53}$% $562\frac{13}{177}\frac{28}{7}$%. **8** *a* $12\frac{1}{2}$%; *b* $11\frac{2}{3}$%; *c* 33%; *d* $12\frac{1}{2}$%; *e* $7\frac{1}{2}$%; *f* 88%; *g* $12\frac{1}{2}$%; *h* 20%; *i* $\frac{1}{2}$%; *j* $14\frac{1}{4}$. **9** *a* 6%; *b* 30%; *c* $7\frac{1}{2}$%; *d* 5%; *e* 50%; *f* 16%; *g* 4%; *h* $15\frac{65}{869}$%; *i* 15%; *j* $14\frac{2}{7}$%. **10** $6\frac{2}{3}$% $26\frac{2}{3}$% 200% 800% $13\frac{1}{3}$% $15\frac{25}{75}$% $7\frac{133}{181}$% $11\frac{3}{7}$% $15\frac{366}{1469}$% $6\frac{8}{82}\frac{12}{43}$%. **11** 50% 35% 28% $804\frac{22}{87}$% $93\frac{1}{3}$% 100% $38\frac{3}{5}$% $10\frac{15}{18}$% 5% $7\frac{1}{7}$%. **12** $36\frac{1}{3}$%. **13** Canada, $5\frac{328411}{800000}$%; U.S., $69\frac{731827}{800000}$% $14\frac{2}{7}$%.

57 *Answers:* **1** Mass., .22+% N.Y., 1.36% Cal., 4.45+% Texas, 7.49+%. **2** Moh., 5.8+% Brah., $20\frac{1}{17}$% Bud., 40%. **3** 650%. **4** Col., 13+% Ind., $\frac{3}{4}$+% Whites, 86.1+%. **5** 87.3+%. **6** 1.4+%. **7** 5%. **8** $8\frac{64}{143}$%. **9** 14.2%. **10** 15%. **11** 19.2%. **12** $1\frac{1}{4}$%.

Notice the wording of **3**. Ask the pupils to give examples of both kinds of exercises.

The term base might be introduced here as being the amount on which the rate is estimated. Some introduction will have to be given to show what the base is in such exercises as **7**, **8**, **9**, and **11**.

58 On this page and the two following pages are simple applications of the two principles of percentage already taught. The pupils should perform the problems by analysis, recognizing in the varying conditions what principle is to be applied. Of the two methods of finding the interest of a given sum for a period greater or less than a year, it is better on some accounts to find the interest first for one year, and then for the given time, as indicated in **8**. Let the analysis be simple and direct, no set form being required.

59 By "rate of interest," referred to in **1**, the pupils should know that the rate *per year* is meant. The pupils should be told that the per cent of gain or loss in business transactions is always estimated on the cost unless otherwise specified. Some talk about

insurance, commission, savings banks, etc., will be helpful in connection with problems bringing in those terms. For facts concerning these subjects see Book VII., and accompanying notes in the Manual.

60 Some of these problems will need to be solved by the aid of figures. The analysis of the most difficult ones may be written out if thought best.

61 In these exercises the base is asked for. Analysis in the use of common fractions should first be made, such as: "Four is one half of some number; two halves of the number is two times four, or eight"; and again: "Twenty-four is two thirds of some number; one third of the number is one half of twenty-four, or twelve. If one third of the number is twelve, three thirds of the number is three times twelve, or thirty-six."

From such reasoning the step to similar work in percentage will be easy. The pupils will readily see that $\frac{50}{100}$, or 50%, may be treated exactly as $\frac{1}{2}$ is treated, and that 12 is 50% of 2 times 12. When the required number cannot be obtained directly by multiplying or by using an equivalent common fraction for the given per cent, 1% of the number is first found, and then 100%; thus, in **10**, the pupil will say: "Six dollars is six per cent of some number; one per cent is one sixth of six dollars, or one dollar. If one per cent of the number is one dollar, one hundred per cent of the number is one hundred times one dollar, or one hundred dollars."

A similar analysis should be made in solving all the problems on this page. To assist in making the solution clear, the analysis may be written out; thus, in **13**:

$\frac{40}{100}$ of the number $= 32$
$\frac{1}{100}$ " " " $= \frac{1}{40}$ of 32
$\frac{100}{100}$ " " " $= \frac{100}{40}$ of $32 = 80$
32 is 40% of 80.

62 Use the common fractional form in the solution of **1**; and generally, when the pupils find difficulty in solving a problem in

percentage, let them substitute small numbers in place of large ones, and common fractions in place of percentages. The substitution of aliquot parts should be made for percentages whenever simplicity is gained, but for useful practice some of the percentages on this page should be used; thus, in **4**, the analysis may be: "$\frac{100}{100}$ = the required number; $45 = \frac{90}{100}$; $\frac{1}{100} = \frac{1}{90}$ of 45, or $\frac{1}{2}$; $\frac{100}{100}$ = 100 times $\frac{1}{2}$, or 50." And in **8**: "$\frac{100}{100}$ = the required number. $\$40 = \frac{25}{100}$; $\frac{1}{100} = \frac{1}{25}$ of \$40, or 32¢; $\frac{100}{100}$ = 100 times 32¢, or \$32."

63 *Answers:* **1** $308\frac{1}{3}$ $2155\frac{5}{7}$ \$6493 \$380800 \$12006.25. **2** £42500 $1739\frac{3}{23}$ tons $2409\frac{23}{33}$ tons 365 da. $19405\frac{5}{9}$. **3** \$11552.94 $8183\frac{1}{3}$ \$14028.55+ \$8728.88+. **4** $88\frac{8}{9}$. **5** $214\frac{2}{7}$. **6** 420. **7** $822\frac{2}{9}$. **8** $405\frac{5}{7}$. **9** 345. **10** 105. **11** 10800. **12** 120. **13** $111\frac{1}{9}$. **14** $2741\frac{7}{9}$. **15** 28000. **16** 252. **17** $\frac{1}{38}$. **18** 100. **19** 93.98+. **20** \$2517.482+. **21** \$2500. **22** 1225 bu. $2858.33\frac{1}{3}$. **23** \$113.35. **24** \$265.07. **25** \$155928.88+. **26** $531\frac{1}{4}$. **27** \$4.04, cost per bbl.

The following form of written analysis for these exercises is suggested:

21 $\frac{100}{100}$ = cost of house.

$\frac{116}{100}$ = selling price of house = \$400.

$\frac{1}{100}$ of cost $= \frac{\$400}{116}$.

$\frac{100}{100}$ of cost $= \frac{\$400 \times 100}{116} = \$344.83$.

64 The miscellaneous exercises on this and the following pages of the section involve all the principles of percentage which have been taught. Do not give the pupils any direct assistance in their solution; nor should any assistance be given until the pupils have made a strong effort to do the work by themselves. If the conditions of a problem are not understood, make them clear by judicious questioning.

65 *Answers:* **1** 352.83⅓. **2** $39\frac{8}{13}$. **3** 306. **4** 1.41. **5** 42 lb. 2⅔ oz. **6** 1%. **7** 0.525. **8** 100%. **9** 180%. **10** 0.01665. **11** 4900. **12** 25. **13** $2.247. **14** $4\frac{12}{47}$%. **15** $7317\frac{3}{41}$ men. **16** 3.696. **17** $774\frac{22}{47}$. **18** $55\frac{73}{126}$%. **19** 87½%. **20** $3140\frac{3}{4}$. **21** 5375. **22** 14904. **23** 1100. **24** 8361½ bu. **25** $\frac{1}{180}$. **26** $74\frac{89}{134}$%. **31** $236¼ $1111¼. **32** $8250 $18250. **33** 1¼¢ per lb. $22.50 entire gain 20 lb. for $1.

66 *Answers:* **1** 2634.016. **2** $5.40. **7** $3 40%. **8** 15% gain 15% loss. **9** 6.62+%. **13** $65\frac{19}{133}$¢. **15** $300.

67 *Answers:* **1** $8888⅞ $13333⅓. **2** $36\frac{8}{19}$¢. **3** $5023.14\frac{22}{47}$. **4** Lose $50. **5** $653.33⅓. **6** $129.16⅔.

68 *Answers:* **1** 30%. **2** $4. **3** $13403\frac{8}{13}$. **4** $6.11⅓. **5** $970.31. **6** $160. **7** $10. **8** 2½%. **9** $1400 $7000 $3500 $14000 $9333⅓ **10** $118.81¼. **11** $1237.50. **12** $6000. **13** $2115 pre. $2500, Lon. $3333⅓, Hon. $2000, Boston $2166⅔, N.Y.

69 *Answers:* **1** Sum paid, $7500 greater. **2** $12828.58. **3** $115, pre. **4** $356.25, cost of insurance. **5** $8.10, com. **6** $37, com. $1443, returned. **7** $40 collected 25%. **8** $400, cost. **9** $20, com. **10** $1969.80, cost. **11** $3900, cost. $29¼, com. **12** 9.36%.

70 *Answers:* **1** 60%. **2** 108 lb. 64⅔ lb. **3** $5.80 on $1000 $.0058 on $1. **4** $240. **5** 4000 lb. **7** $82\frac{2}{11}$%. **8** 2000 lb. wood 37½ lb. ashes. **9** 4400.97+ lb. **10** 83⅓%. **11** $65\frac{3}{10}$%. **12** $56. **13** 6%. **14** $672 left $11.76, int.

71 Four of the most important kinds of business papers are presented in this section, and the teacher should in as practical and attractive way as possible lead the pupils to be familiar with their form and to be able to write them without assistance. Transactions which are most likely to occur, and which involve the making of bills, promissory notes, receipts, and personal accounts, should be given in addition to those here given. Printed blanks

may be provided for the pupils to fill out. The pupils, however, should not depend upon these forms, but learn to write them without aids of any kind. It will be noticed that two forms of bills are given, either form being allowable, except for bills for services, which should be in the form given on page 72, except that the word "For" instead of "To" may be used if desired.

72 *Answers:* **2** Balance due, $186.40. **3** Balance due, $2.68.

73 *Answers:* **1** Amount of receipt, $153.75. **4** Balance, $9.89.

75 *Answers:* **2** Balance due from R., $1.91. **3** Balance due to H., $0.26.

76 *Answers:* **1** Amount due Jan. 24, 1894, $177.63. **4** Amount of bill, $346.55 Cash payment, $169.81. **5** Balance due Parker, $5.02 Balance due Dexter, $1.39 Balance due the merchant, $2.59.

Explain in **4** the custom of giving credit, that is, of giving a certain time in which payment may be made. Also explain the custom of making a discount when money is paid before it is due.

It would be interesting for members of the class to open an imaginary account with each other, using slips of paper for money, and making out the necessary papers that pass between them.

77 *Answers:* **1** $7.00. **2** $76.50. **3** $34\frac{2}{3}$ doz. **4** $138\frac{1}{14}$ yd. **5** 3 da. **6** $11\frac{1}{4}$ da. **7** $2.70. **8** 1079 pesos. **9** $669. **10** Mobile, 92304000 cu. in. 399584+ gal.; St. Paul, 40032000 cu. in. 173299− gal.; Boston, 66816000 cu. in. 289247− gal.; Chicago, 46368000 cu. in.; 200727+ gal. **11** $27\frac{1}{2}$ lb.

Several of these problems should be performed orally. **6** will doubtless cause some difficulty. Ask such questions as the following if necessary: "At the end of three days how many days would it take 9 men to finish it? Suppose all had left but one man, how many days would it have taken to finish the work? If it takes one man so long, how long would it take 4 men?" Give similar exercises varying the conditions.

78 *Answers:* **1** 19+ da. $19.96. **2** 43 bbl. $166\frac{1}{18}$ lb. **3** $153. **4** $324.63. **5** $140.63. **6** $27.34. **7** $43.20. **8** $\frac{1}{4}$ of a pint. **9** $54.085. **10** $26\frac{2}{15}$ bags. **11** £16 8*s* (9*d*). **12** $3430. **13** 119 T. 1531 lb. **14** $21.78. **15** $141.86. **16** $194\frac{13}{28}$ ft. **17** 8 ft. wide.

The 6 mo. in **13** is to be regarded as $182\frac{1}{2}$ days. **17** is a practical problem in finding the greatest common divisor.

79 *Answers:* **1** 6 A. 74 rd. 93.5 sq. ft. **2** $\frac{51}{80}$. **3** $1.52 18.18\frac{2}{3}$. **4** $\frac{231}{256}$ cd. $5.188+. **5** $67\frac{5}{7}$ gal. $562\frac{1}{2}$ lb. **6** 18.487\frac{1}{2}$. **8** 50.2656 sq. rd. .31416 A. **9** $12\frac{8}{25}$ cu. ft. **10** 250 lb. **11** $1333\frac{1}{3}$ lb. **12** 1130.976 cu. ft. **13** 21120^{1} 211.2^{hl} 21120^{Kg}. **14** $41.60 $78.40.

In **7**, counting Sundays, the answers would be: 40033200 copies, and $800664; not counting Sundays, the answers would be: 34329840 copies, and $686596.80.

80 *Answers:* **1** $1462.50. **2** $37.50 per mo. **3** $571.10. **4** $1.74. **5** 63 bu. **6** $198.45. **7** $1\frac{23}{33}$ mi., or 1 mi. 223 rd. $\frac{1}{2}$ ft. **8** $17. **9** $138\frac{2}{15}$ bu. **10** 840 lb. **11** $320 $9120. **12** 405 steps. **13** $11\frac{1088}{2471}$ Ha. $381\frac{647}{7413}$. **14** $8.802\frac{1}{2}$. **15** $1152.

81 *Answers:* **1** 63.1504+ bu. **2** 51.130+ bu. **3** 88 furrows $\frac{511}{660}$ A. **4** $12 less. **5** $2398.50. **6** $4000. **7** $27\frac{81}{125}$. **8** 133.67+ oz. 33.41+ oz. 16.7+ oz. **9** $10. **10** $800. **11** $.02 80%. **12** $7\frac{1}{2}$ m. past 2 P.M. **13** 41 yd. 15 in. **14** 2.581 m. **15** $36666\frac{2}{3}$.

The answers to **8** are found on the supposition that 1 cu. ft. of water weighs 1000 oz. Let the pupils perform the problem, reckoning 1 lb. of water as measuring 27.7 cu. in. This is a more accurate measurement.

82 *Answers:* **1** $21600. **2** $28.95. **3** $.12 per gal. **4** $15.39. **5** 10000. **6** 3200 lb. **7** $5333\frac{1}{3}$ lb. **8** $36\frac{2}{3}$ cu. ft. $5\frac{223}{1000}$ T. **9** 124.2 T. **10** $131\frac{3}{17}$ panes 132 posts. **11** 20106.24 sq. rd. 8293.824 ft. **12** $63. **13** $1080.

83 *Answers:* **1** 75¢. **2** 100%. **3** \$133⅓ 26⅔. **5** 300 sq. ft. **6** \$16000. **7** 560.2+ rev. **8** $9\frac{1}{51}$ $60\frac{20}{153}$. **9** 118 qt. boxes. **10** 80 rails 4224 rails. **11** 173 pills. **12** \$1 per crate. **13** \$24000. **14** \$33.68. **15** \$358.06+ $6\frac{120252445760}{}$% \$53369557.71.

Physicians' prescriptions are frequently made in the manner indicated in **11**. The given quantity should be read 8 drams and 2 scruples.

84 *Answers:* **1** \$10440.80. **2** 969⅛ bu. **3** \$1.80. **4** Monday Wednesday. **5** \$1.15 gain. **6** $\$3.38\frac{23}{71}$. **7** \$1620. **8** 10890 sq. ft. 121 ft. **9** \$8.12½. **10** 800 da. 333⅓ da. **11** Cheaper by the dozen \$1 cheaper. **12** $13\frac{2}{5}\frac{1}{4}\frac{1}{4}\frac{7}{5}$ A. **13** \$4142 gain.

85 *Answers:* **1** 864 books. **2** 6 widths 30 yd. 7 widths 37⅛ yd. **3** Apr. 10, 1876. **6** 15 lb. 1 gal. 1 gi. **7** 3 oz. 6 oz. **8** 493⅛ ft. $8\frac{4}{37}$ mi. **9** 8 lots. **10** 80 doses. **11** 88 bottles. **12** $14\frac{1}{32}$ yd. **13** \$2.19. **14** 205 pills. **15** 22.79 bbl. **16** 86.78+ bu.

86 *Answers:* **1** \$672. **2** \$480. **3** 1215 cu. ft. **4** 243⅛ yd. **5** $77\frac{19}{25}$ A. **6** 48 rd. **7** \$163.74+. **8** $13\frac{5}{8}\frac{9}{4}$ cd. **9** 27 b. 20 b. **10** 12 da. 4 da. **11** 136 men. **12** 1½ bu. **13** ⅓. **14** \$30000 \$64000. **15** \$3.46. **16** $\$1309\frac{1}{11}$.

87 *Answers:* **1** 19712 bricks 14601+. **2** 316⅔ rev. $34\frac{1}{11}$ mi. **3** 396 tiles. **4** ¼% of 5000. **6** 55⅝%. **7** Chicago ahead $2\frac{11}{12}$%. **8** \$2812.50. **9** $11\frac{11}{16}\frac{3}{4}$¢. **10** $40\frac{13}{30}$ yd. **11** $\$14.91\frac{3}{4}$. **12** \$68.40. **13** $1\frac{7}{30}$ da. **14** \$30000 D \$2373⅛ E. **15** 960 steps. **16** \$4118.40.

88 *Answers:* **2** $\frac{17}{19}\frac{8}{9}$ (whole, 3184). **3** $1\frac{47}{152}\frac{9}{1}$ ft. $2\frac{1}{11}\frac{6}{7}$ ft. **4** 50166869 62633336 42.09%. **6** About $\frac{1}{18}$ About 2½ times. **7** 389+ times. **8** 4.38% 9.05%. **9** 11.65%. **10** 16.22%. **11** 28.16 gal. **12** 420^{Kg}. **13** $2017888\frac{8}{9}$ bbl. **14** $189177083\frac{1}{8}$ bu.

89 The answers to these exercises will be approximate only, and will therefore vary greatly. As the purpose of the work is more to get the pupils to measure on the map and to learn valuable information in geography than to acquire arithmetical knowledge and skill, close, accurate work should not be insisted upon. Let the pupils measure by the aid of the scale of miles to which the map is drawn, and also by the aid of parallels and meridians.

90 *Answers:* **1** 32628.1824 lb. 13111.9452 lb. **2** .23998. **3** 294148.008 lb. 374520 lb. **4** 588060 lb. **5** 30000 lb. **6** 3 oz. 17 pwt. 12 gr. **7** 34 ft. **8** 339.87^{m}. **9** 1699.35^{m} or 337.89257 rd. **10** 1 mi. 237.52+ rd. **11** 37.88 sec. **12** 202.73 rds.

SECTION IX.

NOTES FOR BOOK NUMBER SEVEN.

For general suggestions as to the kind of work peculiar to advanced grades in Arithmetic, teachers are referred to the "Note to Teachers" on pages iv and v, particularly to what is said of rules, definitions, and explanations of problems. In these forms of expression special attention should be given to accuracy of statement, and by accuracy is meant not merely a close adherence to any one particular form, but an exact expression of a logical process. Loose statements which earlier in the course were allowed to pass should now be challenged and corrected. The pupils should be led by judicious questioning to detect inaccuracies of their own as well as of others' statements.

A rule in Arithmetic is a general statement of the method of solving a given class of problems. A rule should therefore be derived from the process of solution, and, as far as possible, be made by the pupils themselves. The rule thus made by the pupils may be amended by means of criticism and comparison with a correct form.

A definition is a general statement of the peculiar and characteristic properties of the subject defined. An analysis of the subject to be defined should be made, and from the analysis the definition should be deduced, if possible, by the pupils, with such correction as they may be able to make. For example, if it is desired to teach the definition of addition, the teacher should lead the pupils to perform or to recall the process of addition, and in that process to see that they have found a number which contains as many units as all the given numbers taken together. After being told that this process is addition, the pupils will say: "Addition is the process of finding a number which contains as many units as all the given numbers taken together." On being told that the number found is called the sum, they will correct the definition given, and say: "Addition is the process of finding the sum of two or more numbers." If this definition is still thought to be incomplete, attention may be called by examples to the fact that only numbers of the same kind can be added, as, 8 pounds and 6 pounds. 8 pounds and 6 ounces might be put together to express a denominate number, as 8 lb. 6 oz., but such putting together is not addition. The pupils will therefore further correct their definition, and say: "Addition is the process of finding the sum of two or more numbers of the same kind."

An explanation of a problem should be made in such a way as to be easily understood, and be so comprehensive as to include a statement of the process of solving the problem and reasons for each step of the process. It is not advised that all or many of the problems performed be explained, but as many of each class of problems should be explained as to give assurance that they are thoroughly understood. In this and the following book it is recommended that the processes only of many of the problems be indicated, either orally or in writing, it being assumed that the mere computations may be accurately performed.

In the work of previous books the attention of teachers has been called repeatedly to the importance of illustrations by plans or diagrams. The illustrations should be continued in connection

with all difficult problems; but for the solution of problems not very difficult the pupils should not depend upon such helps, but imagine and state the conditions as nearly as possible. If, however, their statements are questioned or are not understood, they should be ready to make the needed illustrations.

1–6 The solution of these review problems will give the pupils little difficulty, if the previous work has been well done. Whenever difficulty is found, lead the pupils by questions to see exactly what is required, and what steps are necessary in the solution. For example, if the pupils find difficulty in performing **7**, page 3, ask the questions: "Before you can find the worth of the remainder, what must you find? How can that be done?" If the pupils choose to find in a roundabout way the number of gallons remaining, let them so find it; but after it is found ask them if there is not a shorter way. Some bright pupil will see that 26 qt. $= 6\frac{1}{2}$ gal., and that this added to $13\frac{1}{4}$ gal. is $19\frac{3}{4}$ gal., which subtracted from $28\frac{1}{2}$ gal. gives $8\frac{3}{4}$ gal. In such exercises as **8**, page 3, let the pupils make a diagram before the solution is made.

7 *Answers:* **1** $\frac{5}{32}$ $\frac{15}{176}$ $\frac{5}{44}$ $\frac{103}{1760}$ $\frac{189}{528}$ $\frac{7}{880}$. **2** $32 $184.73 $5.94 $140.04. **3** $52.50. **4** 2 rd. 3 yd. **5** 76 yd. 8 in. **6** 155 rd. $1\frac{1}{2}$ ft. $927.61 $1902.27. **7** 10 yd. **8** 80 rd. 120 rd. 200 rd. 288 rd. **9** .11875 mi. .16098+ mi. **10** $1760. **11** $389.78 $406.74. **12** 120 cups. **13** 22 bu. 3 pk. **14** $\frac{5}{121}$ $\frac{15}{121}$ $\frac{17}{32}$ $\frac{3427}{8712}$ $\frac{385}{484}$. **15** $50 $112.50 $9.15 $185.38.

In the solution of **11**, lead the pupils to find the number of cubic inches or feet that a cental or hundredweight occupies. One bushel, or 60 lb., occupies 2150.4 cu. in.; 100 lb. would occupy $\frac{100}{60}$ of 2150.4, or 3584 lb. The solution of the first part of the exercise would be indicated thus:

$$\$1.15 \times \left(\frac{37 \times 19 \times 6 \times 1728}{2 \times 3 \times 3584}\right)$$

8 *Answers:* **1** 201 A. 20 sq. rd. **2** 11 A. 55 sq. rd. $181\frac{1}{2}$ sq. ft. **3** 5 A. 17642.7+ sq. ft. **4** $837\frac{9}{13}$ ft. **5** 50.26+ sq. ft. **6** 12.7+A. **7** $1360. **8** 926.25[1]. **9** $15200. **10** (*a*) $524.38; (*b*) $2855.25;

(*c*) 159 posts; (*d*) \$57.04; (*e*) \$31.07; (*f*) $1027\frac{1}{27}$ cu. yd. **11** 36 ft. **12** 5624.64 oz. **13** $5\frac{85}{108}$ cu. ft. 1296 cu. ft. **14** $11\frac{1}{9}$ cu. yd. $1\frac{209}{2916}$ cu. yd. **15** $3\frac{45}{128}$ cords. **16** 41 ft. $2\frac{8}{81}$ in.

9 *Answers:* **1** (*a*) 519 sq. ft.; (*b*) $177\frac{7}{9}$ sq. ft.; (*c*) $19\frac{81}{81}$ sq. yd. (*d*) \$16.54; (*e*) $38\frac{4}{9}$ sq. ft.; (*f*) across the room, widthwise. **2** $99\frac{11}{32}$ cu. ft. **4** 6 ft. **5** $6\frac{15}{64}$ cords. **6** $1615\frac{81}{77}$ gal. 180 sq. ft. **7** \$560 \$120. **8** 18150 cu. ft. \$2637.42. **9** \$264.

10 *Answers:* **1** 4800. **2** 35.2+ qt. **3** $115\frac{3}{8}\frac{7}{7}\frac{3}{2}$ cu. ft. **4** \$2.69 \$1.35 \$.68 \$7.35 \$24. **5** \$3 \$3.06 \$2.76 \$2.45. **6** $130\frac{2}{3}$ loaves 45.9+%. **7** $166\frac{2}{3}$% \$.093$\frac{3}{4}$. **8** \$14.24. **9** 828.8 T. 417.2 T. **10** \$5180. **11** 405 bu. **12** $3\frac{1}{14}\frac{1}{8}$. **13** \$40500. **14** $\frac{38}{195}$ $\frac{187}{195}$ \$44.60.

11 *Answers:* **1** Amt. of bill, \$383.20. **2** \$6120. **3** $12\frac{1}{2}$%. **4** $46\frac{2}{3}$% lost. **5** $120\frac{8}{33}$ rd. **6** 1 h. 6 min. 9 sec. **7** \$418 \$33.589+. **8** $4\frac{1}{2}$%. **9** $73\frac{1}{2}$ planks. **10** \$19.125 \$.149+. **11** $220\frac{4}{27}$ yd. \$326.11. **12** \$1743.09.

12 *Answers:* **1** $1\frac{2}{3}\frac{4}{3}\frac{8}{7}$ da. **2** \$4876. **3** $5\frac{1}{3}$ mo. **4** \$18.375. **5** 2.75. **6** \$14375. **7** 25.43+ sec. **8** $33\frac{1}{3}\frac{3}{3}$ T. \$67.86. **9** $567\frac{7}{9}$ cu. yd. **10** $22\frac{8}{8}\frac{6}{7}$%. **11** 180 mi. **12** 529.2 lb. **13** \$64 \$14.80. **14** \$1.125 \$3.60 \$2.30. **15** \$44.84.

13 *Answers:* **6** $28\frac{4}{5}$¢. **7** $9\frac{1}{5}$¢. **8** \$15.64 \$21.16. **9** \$69.12 \$453.12.

Before giving these exercises in Profit and Loss, it might be well to tell the pupils something of the common practice of merchants in receiving a profit for the trouble of buying and selling goods, and to give a few examples.

14 *Answers:* **5** $33\frac{1}{3}$% $33\frac{1}{3}$%. **6** $11\frac{1}{9}$%. **7** $11\frac{1}{9}$% on sugar $16\frac{2}{3}$% on kerosene. **8** $77\frac{7}{9}$%. **10** $22\frac{2}{9}$%. **11** $14\frac{2}{17}$%.

Lead the pupils to see that the per cent of gain or loss depends not only upon the amount of gain or loss, but also upon the cost. This is illustrated in **5**. In **9**, the supposition is that the part unsold is worth $\frac{1}{4}$ of what was paid for the farm. In this case the

answer would be 25%. Ask the question: "What was the per cent of gain on the part sold ($33\frac{1}{3}\%$)?" If necessary, lead the pupils to use numbers as \$2000, for the cost of the farm. The cost of the part sold would then be \$1500; and if it were sold for \$2000, the gain would be $33\frac{1}{3}\%$. Again ask: "What would be the per cent of gain if I should sell the other fourth at the same rate?"

After the pupils are familiar with the process, the common fractional part of the cost may be omitted, the per cent of cost being found directly.

15 *Answers:* **4** $16\frac{2}{3}¢$ $3\frac{1}{3}¢$. **5** \$40. **6** \$2.50. **7** \$3.75. **8** \$90. **9** \$5 \$5.75. **10** $\$4\frac{8}{9}$. **11** Lost $\$26\frac{2}{3}$. **12** \$16304.35. **13** $\$1.07\frac{1}{2}\frac{9}{5}$.

In explaining these problems, the pupils should begin with the statement that the cost is called for, and that it equals $\frac{1}{1}\frac{0}{0}\frac{0}{0}$ of itself. The rest of the explanation may be given as indicated in the written analyses, the reason for each step being added; thus, in **3**: "\$360 = selling price = $\frac{1}{1}\frac{2}{0}\frac{0}{0}$ of cost. $\frac{1}{1}\frac{}{0}\frac{}{0}$ of cost = $\frac{1}{1}\frac{}{2}\frac{}{0}$ of \$360 or \$3. \$3 = $\frac{1}{1}\frac{}{0}\frac{}{0}$ of cost. $\frac{1}{1}\frac{2}{0}\frac{0}{0}$ of cost = 100 times \$3 or \$300."

After the pupils are familiar with the solution of this kind of problems, the use of hundredths as given in the analysis may be omitted, and simpler fractions may be substituted; thus, in **11** the solution might be:

\$200 = 125% or $\frac{5}{4}$ of cost of 1st horse.
$\frac{4}{5}$ of \$200 = \$160 = cost of 1st horse.
\$200 = 75% or $\frac{3}{4}$ of cost of 2d horse.
$\frac{1}{3}$ of \$200 = $\$266\frac{2}{3}$ = cost of 2d horse.
$\$66\frac{2}{3}$, loss on 2d horse — \$40 gain on 1st horse = $\$26\frac{2}{3}$ = net loss.

Problems may also be performed by a short process on a line; thus, in **12**:

$$\frac{\$10000 \times 100 \times 3}{92 \quad \times 2} = \text{cost of ship.}$$

Pupils should always be able to explain problems performed in such a way. The explanation of the above solution would be, "If

the loss was 8%, the selling price would be 92% of the cost. \$10000, the selling price = 92% of the cost of $\frac{3}{8}$ of the vessel. $1\% = \frac{1}{92}$ of \$10000, found by dividing by 92, and $100\% = 100$ times this quotient. $\$10000 \div 92 \times 100 =$ cost of $\frac{3}{8}$ of the vessel. $\frac{1}{8}$ of the vessel cost $\frac{1}{2}$ of this sum, found by dividing by 2, and $\frac{3}{3}$, or the cost of the vessel, is 3 times this quotient."

16 *Answers:* **6** 25% gain. **7** $86\frac{1}{4}¢$. **8** 30¢. **9** 72 baskets. **10** \$3 $\$2.72\frac{8}{11}$. **11** $48\frac{48}{53}\%$ gain.

Lead the pupils to define these terms in the manner indicated on page 140 of the Manual. The following definitions may assist teachers in securing concise and comprehensive statements:

The *base* in Profit and Loss is the number of which the per cent of gain or loss is to be found. It is usually the cost.

The *rate per cent* is the number of hundredths that is taken of the base.

The *percentage* is the number found by taking a certain per cent of the base. The percentage in Profit and Loss is the amount of gain or loss.

The *amount* is the number which includes the base and percentage, and is the selling price in Profit and Loss when the percentage is the amount of gain.

The *remainder* is the difference between the base and percentage, and is the selling price in Profit and Loss when the percentage is the amount of loss.

The rules called for in **3** to **5** should be deduced by the pupils from the corresponding processes, and may be as follows:

To find the gain or loss, multiply the cost by the rate per cent.

To find the rate per cent, divide the gain or loss by the cost, carrying the division to hundredths.

To find the cost, divide the selling price by 100, increased by the rate per cent of gain, or decreased by the rate per cent of loss, and multiply by 100.

17 *Answers:* **1** \$11095.20 $154\frac{1}{10}\%$. **2** $68\frac{8}{19}\%$ $40\frac{5}{8}\%$ $5\frac{5}{19}\%$. **3** $4\frac{8}{23}\%$ loss. **4** \$1820 30%.

Some of the exercises from **5** to **20** will have two sets of answers according as the percentage is gain or loss. Let the pupils make these problems, having the percentage either profit or loss as they choose.

18 The explanation included in **1** may have to be enlarged or extended by the teacher. Possibly other examples than these given will assist the pupils to understand more fully the business of commission. Besides giving the names of the consignor and consignee, the pupils should be led to answer in general that "the *Consignor* is the person who sends the goods to another," and "the *Consignee* is the person to whom the goods are sent."

The person for whom a buyer or seller of goods acts is the *Principal.*

The person who is authorized to buy or sell goods for another is called an *agent.*

The allowance made to an agent for buying and selling goods is the *Commission.*

Attention should be called to the fact that the base in commission is the cost or selling price, according as the agent is a buyer or seller.

19 *Answers:* **3** \$840. **4** \$2500.00. **5** \$3947.33. **6** \$49.07. **7** $2\frac{1}{2}\%$. **8** \$2125. **9** \$5625.20. **10** \$27.60. **11** \$880. **12** 10279.50.

The pupils should be able to tell after reading over a problem whether the base is given or not. Custom varies somewhat as to what the base should be for estimating the commission in case of a selling agent who pays expenses of cartage, storage, etc. When not otherwise specified the selling price is regarded as the base.

20 *Answers:* **4** \$45. **5** \$75 \$2075 (if premium is counted). **6** \$1500. **9** \$79.50. **10** \$2637.50 more. **11** \$47.93.

Bring to the class insurance policies of various kinds. Blanks of policies may be procured from insurance agents, or policies owned by the parents of pupils may be borrowed. Explain all the terms that are used in the policies, and the purpose and plan

of carrying on the business of insurance. The methods of organizing a joint stock company and some features of such a company are shown on pages 49 and 50, Book VII. Teach, as previously shown, the following definitions:

A *policy* is a written contract, insuring security against loss between two parties.

The *premium* is a sum paid for insurance.

The *underwriter* is the insurer.

Refer to the fact that mutual companies usually charge a higher rate of insurance than stock companies, but that the excess is offset by dividends declared. Give problems involving insurance in both kinds of companies to ascertain which would involve the least cost, taking into account the interest of the excess paid to a mutual company.

21 *Answers:* **1** $\frac{2}{3}\%$. **2** \$1936. **3** \$40. **4** \$241.13. **5** \$2666$\frac{2}{3}$. **6** \$2842. **7** \$30000. **8** 2%. **9** \$1000. **10** \$2500. **11** \$23985. **12** \$1675. **13** \$5 less.

22 *Answers:* **3** $1\frac{1}{4}\%$ \$.0125. **4** \$88.80. **5** \$12 \$.012 \$40.80. **6** $1\frac{1}{12}\frac{8}{25}\%$ \$74.56. **8** \$16.912 \$718.76.

In the preliminary teaching lesson, use familiar examples, both in showing the purpose of taxation and the meaning of the various terms used. From recent reports of assessors, give actual facts of the valuation and rate of taxation of the city or town in which the pupils live.

Real estate is fixed property, such as lands and houses.

Personal property is property that is movable, such as money, stocks, cattle, furniture, etc.

A *poll tax* is a tax upon the person of a citizen.

Assessors are officers whose business it is to estimate the value of property, and to apportion among individuals the sum to be raised.

In finding the method of assessing a tax, get from the local assessors the method that is actually employed by them. In some places the assessors are paid a regular salary, and in assessing the

tax their salaries are not taken into account. The amount of tax uncollectible cannot be known before the tax is collected, but frequently allowance is made, based upon the experience of past years. The following is a practical rule for finding the rate of taxation where the overlay is not more than 5% of the sum to be raised on the property:

Divide the net sum to be raised by 96, and multiply the quotient by 100. From this sum subtract the poll tax, and divide the difference by the amount of taxable property. If the overlay is greater than 5%, the first divisor must be correspondingly less.

23 *Answers:* **2** \$52.50. **3** \$53.40. **4** \$134.10. **5** \$18.75. **6** \$98.70. **7** \$212763.72. **8** \$4500.

In **7**, deduct the uncollectible tax before finding the cost of collecting or net proceeds.

24 *Answers:* **3** \$188.31. **4** \$46.19. **5** \$100. **6** \$656.64. **7** \$338.62. **8** \$145. **9** \$42.

In teaching this subject show the reasons which governments have for taxing imported goods. The following facts might be presented in one form or another:

In addition to the usual duties collected at the custom-house, a tax called *tonnage* is levied upon vessels according to the number of tons they can carry. In some places duties are paid upon articles of export. An invoice of goods, called also a manifest, sometimes gives a wrong valuation. To guard against this fraud, a certificate from the consul is sometimes required, stating that the prices given in the invoice are the prevailing market prices. The terms *leakage* and *breakage* are used to denote the allowance for waste of liquids and for the breaking of bottles. Unless otherwise specified, the value of a pound sterling in U. S. money is reckoned at \$4.866.

25 *Answers:* **1** \$888. **2** \$17.53. **3** \$18.19. **4** \$2881.25. **5** \$97.18. **6** \$1687.50. **7** \$2,13. **8** \$252.76. **9** 300 lb. **10** \$45,68.

26 *Answers:* **1** \$40.70 \$2679.30. **2** 4325 bu. **3** \$451.50. **4** \$405292.50 \$371518.12½. **5** \$60. **6** \$500 loss. **7** \$2209.80. **8** \$1.008. **9** 1%. **10** \$12466.02. **11** \$6155.85.

In **8**, if 2240 lb. are taken as a ton the answer is 90¢.

27 *Answers:* **1** \$326.40 loss. **2** \$3.323. **3** \$.003473 \$46.65. **4** \$29.62. **5** \$2866.20. **6** \$838.50. **7** \$659.50. **8** 1% 3% ⅝% **9** \$1900.59. **10** 18 machines.

28 *Answers:* **1** 1½%. **2** \$1680.22. **3** Silk @ \$5.57 Ribbon @ \$.295 Lace @ \$1.03. **4** \$5866⅔. **5** \$644.32. **6** \$18625. **7** 1½%. **8** \$853.13. **9** A, \$20.51 B, \$23.67 C, \$42.60 D, \$12.62. **10** \$695.76. **11** \$5585.08.

29 These problems should be performed orally and by the shortest way. Where the way is not indicated in the question, let the pupils choose the method of solution.

30 *Answers:* **4** .142. **5** .091. **6** .1815. **7** .036⅓. **8** .0715. **9** .097. **10** .041⅔. **11** .063. **12** .036⅔. **13** .135⅛. **14** .062½. **15** .037½. **16** .211⅛. **17** .246½. **18** .093⅛. **19** .279. **20** .127⅔. **21** .2285. **22** .362⅝. **23** .178⅛. **24** .109⅜. **25** \$18.67. **26** \$43.05. **27** \$33.37. **28** \$38.15. **29** \$180.49. **30** \$23.75. **31** \$24.08. **32** \$109.12. **33** \$37.62. **34** \$20.05. **38** \$411.25. **39** \$278.85.

While occasionally the method of casting interest is determined by peculiar conditions, it is generally best to adopt and follow one method. A method very generally pursued by business men is what is called the 60-day method. By this method the interest is ascertained first for 60 days by getting $\frac{1}{100}$ of the principal, and for other times from this sum. For example, if it is required, as in **25**, to find the interest of \$200 for 1 yr. 6 mo. 20 da. at 6%, first divide by 100 to find the interest for 2 mo., and multiply that sum by 9 to find the interest for 18 mo. 20 da. is ⅓ of 60 da., and therefore divide the interest for 2 mo. by 3. Add results for the final result. The solution will appear as follows:

$200.00	Principal.		
2.00	Int. for	2	mo.
18.00	" "	18	mo. (2 mo. × 9)
.67	" "	20	da. ($\frac{1}{3}$ of 60 da.)
$18.67	" "	18	mo. 20 da.

Not all problems can be so easily performed by this method, but a little practice will give the pupils skill in finding the desired fractional parts. Thus, in **30**, find the interest for 2 mo., then for 15 da. by dividing the interest for 2 mo. by 4, then for 2 da. by dividing the interest for 60 da. by 30 ($\frac{1}{8}$ of $\frac{1}{10}$). In **33**, multiply the interest for 2 mo. by 2 to find the interest for 10 mo. Divide the interest for 2 mo. by 2 to find the interest for 1 mo. Get $\frac{9}{10}$ of the interest for 1 mo. to find the interest for 27 da. This solution will appear as follows:

Interest of $632.29 for 11 mo. 27 da.?

	6.322	Int. for	2 mo.
	$31.61	Int. for	10 mo. (2 mo. × 5).
.316	3.16	" "	1 mo. ($\frac{1}{2}$ of 2 mo.).
9	2.84	" "	27 da. ($\frac{9}{10}$ of 30 da.).
2.844	$37.61	" "	11 mo. 27 da.

When this method is familiar to the pupils, only the times need be written after each partial interest. The following solutions, furnished by business men, illustrate this method of casting interest actually used in business:

Int. of $4120 from Aug. 26, '90, to May 27, '92, @ 5%.

21 mo. 1 da. @ 6% on $4120.

$\frac{1}{2}$% per mo.	20 mo.	412.
	1 mo.	20.60
	1 da.	.68
		433.28
	less $\frac{1}{6}$	72.21
		$361.07

Int. of $2907 from Dec. 2, '90, to May 27, '92, @ 5%.

17 mo. 25 da. @ 6% on $2907.

	16 mo.	232.56
	1 mo.	14.54
	20 da.	9.69
	3 da.	1.45
	2 da.	.97
		259.21
less $\frac{1}{6}$		43.20
		$216.01

The above method is a slight modification of what is called the six per cent method. By this method the interest of \$1 for the given time at 6% is multiplied by the number of dollars in the principal, and if the rate is other than 6%, the interest found is increased or diminished as required. The interest of \$1 for 1 yr. at 6% is \$.06; for 2 mo., \$.01; for 1 mo., \$.005; for 6 da., \$.001; and for 1 da., $\frac{1}{6}$ of a mill. In general, the interest of \$1 is found by taking six times as many cents as there are years, one-half as many cents as there are months, and one-sixth as many mills as there are days. By a little practice one is able to give the interest of \$1 at 6% for any time very quickly, and if the 6% method is pursued it will be good practice for pupils to do this.

31 *Answers:* **1** \$213.33. **2** \$111.88. **3** \$61.25. **4** \$20.35. **5** \$29.89. **6** \$66.72. **7** \$46.39. **8** \$8.15. **9** \$19.70. **10** \$138.03. **11** \$8.59. **12** \$25.95. **13** \$792.25. **14** \$868.49. **15** Apr. 10 and Oct. 10 of each year to Oct. 10, 1893, each payment being \$43.75. The last payment, \$1793.75, including principal and six months' interest, was made Oct. 10, 1893. **16** \$250.30. **17** \$64 \$.175+ \$12.80 \$25.95. **18** \$8.17. **19** \$20.96. **20** \$35.55. **21** \$34.14.

In finding the time between two dates for business purposes, there is a difference of practice among business men. A common practice in estimating time for casting interest, is to regard each month as consisting of 30 days; thus, in finding the time from Sept. 20, 1891, to Aug. 1, 1892, it is 11 months to Aug. 20, and to Aug. 1, it is 19 days less than 11 months, or 10 mo. 11 da.

At the time of purchase (Ex. **15**), Mr. Brown received from Mr. Smith a deed of the property, and at the same time gave to Mr. Smith two papers signed by him, one a note promising to pay Mr. Smith \$1750 with interest at 5% payable semi-annually, and the other a deed, called a mortgage deed, providing that the property shall come into full possession of Mr. Smith in case the terms of the note are not fulfilled. This deed must be acknowledged before a Justice of the Peace or Notary Public.

To find the accurate interest, find the interest for the exact number of days, regarding 365 days as the year. The following solution of **18** will illustrate the method:

30 Find the accurate interest of \$540 for 92 da. at 6%.

31

30

1

92

$$\frac{\overset{108}{\cancel{\$540}} \times .06 \times 92}{\underset{73}{\cancel{365}}} = \$8.166+$$

```
      108
      .06
     6.48
       92
    12 96
   583 2
73) 596.16 (8.166+        8.17 Ans.
    584
     121
      73
      486
      438
       480
```

To find the exact number of days between July 1 and Oct. 1, lead the pupils to say: "30 more days in July, 31 in August, 30 in September, and 1 in October. Adding, we have 92 days." The explanation of the problem is simply: "Multiply \$540 by .06 to get the interest for 1 year. Divide this product by 365 to find the interest for 1 day, and multiply by 92 to find the interest for 92 days."

32 *Answers:* **3** \$15.30 4% 6.84+%. **4** 6% 5% $3\frac{5}{9}$+%. **5** $4\frac{1}{2}$% $5.61\frac{1}{9}$%. **6** 4%. **7** $13\frac{1}{3}$+%. **8** $4\frac{1}{7}$%. **9** 6.53+%. **10** 6%. **11** $4\frac{16}{41}$%. **12** 3.45+%. **13** 3.41+%. **15** \$210. 5 yr. **16** 1 yr. 2 mo. 12 da. **17** 1 yr. 3 mo. **18** 9 mo. 22 da. **19** 1 yr. 7 mo. 1 da. **20** 5 mo. 9 da. **21** 1 yr. 6 mo. 21 da. **22** 2 yr. 5 mo. 8 da. **23** 4 mo. **24** 3 yr. 3 mo. 17 da. **25** 2 mo. 17 da. **26** 4 yr. 8 mo. 20 da. **27** 4 mo. 4 da. **28** 3 yr. 11 mo. 9 da.

Draw reasons as well as results from the pupils, leading them to say in explanation of **1**: "Since at *one* per cent the interest for the given time is \$1 to yield an interest of \$20, it will take as

many per cent as $1 is contained times in $20, which is 20. *Answer*, 20%." The form of explanation may vary somewhat; for example, the reason for dividing may be expressed: "To yield $20 interest the rate must be as many times one per cent as there are times $1 in $20, which is 20."

The form of written solution of this class of problems may be as follows:

6 Find the rate per cent.

$\$480 \times .01 \times 3 = \$2.40$ int. at 1%.
$\$9.60 \div \$2.40 = 4$.
Answer, 4%.

As in finding the rate per cent the interest at one per cent is made the divisor, so in finding the time the interest for one year is used in a similar way. The explanation of this class of problems may be as follows: "Since in *one* year $400 will gain at the given rate $20, it will take as many years for it to gain $60 as $20 is contained times in $60, which is 3. *Answer*, 3 years." The written solution of **17** would appear as follows:

$\$800 \times .05 = \$40$ int. for 1 yr.
$\$50 \div \$40 = 1\frac{1}{4}$.
Answer, 1 yr. 3 mo.

The common custom is to drop the fraction of a day if it is less than $\frac{1}{2}$, and to add a day if it is $\frac{1}{2}$ or more than $\frac{1}{2}$.

33 *Answers:* **3** $266.67. **4** $150. **5** $810. **6** $1425. **7** $2306.49. **8** $1370.30. **9** $3378.64. **11** $4. **12** $16. **13** $620. **14** $1021.504+. **15** Int., $75.95; Amt., $915.95. **16** Time, 6 mo. 12 da.; Amt., $412.80. **17** Rate, $4\frac{1}{2}$%; Amt., $350.60. **18** Prin., $713.958+. **19** Time, 7 mo. 27 da.; Amt., $8700. **20** Rate, 4.39+%; Amt., $1700. **21** Prin., $360.54+; Amt., $387.04+. **22** Int., $13.848; Amt., $294.248+. **23** Time, 9 yr. 2 mo. 10 da.; Amt., $1480.40.

Essentially the same principle is involved in the solution of these problems as in the solution of problems in which the rate or

the time is sought. A principal of *one* dollar is taken when it is desired to find the principal. The following written solutions of **6** and **12** will illustrate a good method:

Find the principal.

$\$1 \times .01 \div 2 = \$.005$ int. of \$1 for 1 mo.
$\$.005 \times 9 = \$.045$ " " " " 9 mo.
$\$.005 \div 5 \times 3 = \$.003$ " " " " 18 da.
$\$68.40 \div \$.048 = \$1425.$

Answer, \$1425, principal.

Find the principal.

$\$1 \times .04\frac{1}{2} \times 2\frac{1}{2} = \$.11\frac{1}{4}$ int. of \$1 for 2 yr. 6 mo. at $4\frac{1}{2}\%$.

$$\begin{array}{r} \$1.1125)\,\$17.8000\,(16 \\ \underline{11\ 125} \\ 6\ 6750 \\ \underline{6\ 6750} \end{array}$$

Answer, \$16, principal.

34 *Answers:* **1** 5 yr. $2\frac{1}{2}$ yr. 3 yr. 1 mo. 15 da. **2** 6%. **3** 1 yr. 8 mo. **4** $16\frac{2}{3}$ yr. 25 yr. $14\frac{2}{7}$ yr. **5** 5%. **6** \$36000. **7** $5\frac{1}{8}\%$. **8** Jan. 1, 1897. **9** \$652.63. **10** May 20, 1894. **11** July 1, 1891. **12** \$2500. **13** \$14400 \$21600. **14** 4%. **15** \$375 loss.

In **15**, by "average expense" is meant average yearly expense.

35 *Answers:* **3** \$100. **4** \$100 cash. **5** \$200. **6** 394.088+. **7** \$345.394. **8** \$177.639. **9** \$1578.947.

No new principle is involved in these problems. The present worth is the principal, which, put at interest, will amount to a given sum in the given time and rate. The explanation drawn from the pupils may be (**3**): "The amount of \$1 for the given rate and time is \$1.05. It will require as many dollars to amount to \$105 as \$1.05 is contained in \$105, which is 100. *Answer:* \$100."

36 *Answers:* **1** \$294.12. **2** \$21.06. **3** \$123.53. **4** The latter offer, \$41.14 better. **5** Less than true value. **6** \$796 \$.02 \$808. **7** \$657.64. **8** \$1188.12 Less \$1211.88+. **9** \$588. **10** \$22.50. **11** \$450. **12** \$745.29.

Let the pupils see, in such problems as **3**, that the answer given is the present gain, the present worth of the note given being regarded as the actual cost in cash.

In finding the true value of the note (**6**) March 1, the principal is found, which, put at interest, would amount to $800 in 1 mo. at 6%. Sometimes more than one discount is made from the list price. Show this to the pupils by giving an instance, as, for example: The list price of tacks is $16 per cwt., the cash price being at a discount of 25% and 10% of the list price, or, as the seller might say, 25 and 10 off. To find the cash price, first deduct 25% from the list price, and then 10% of this result. Thus, 25% of $16 = $4; $16 − $4 = $12; 10% of $12 = $1.20; $12 − $1.20 = $10.80, cash price. It might be well to give several problems of this kind to the pupils.

37 *Answers:* **1** $53.04, due Sept. 1. **2** $44, due Jan. 1, 1894. **3** $218.85.

Pupils should be led to write out fully and clearly an analysis of each problem, and to give an explanation of each item. The following written solution of **2** may suggest a good form for teachers to present as the given questions are answered by the pupils:

Principal, Jan. 1, 1893,	$100.00
Interest to May 1 (4 mo.),	2.00
Amount,	102.00
Payment May 1,	40.00
New principal,	62.00
Interest to Aug. 1 (3 mo.),	.93
Amount,	62.93
Payment Aug. 1,	20.00
New principal,	42.93
Interest to Jan. 1, '94 (5 mo.),	1.07
Amount due, Jan. 1, '94,	$44.00

38 *Answers:* **1** $254.44. **2** $112.81. **3** $255.30. **4** $88.24+. **5** $132.20. **6** $258.42+. **7** $164.29+.

The method above shown of finding the balance due on a note when partial payments have been made, is quite generally followed by business houses. It is in the main the method adopted by the United States Supreme Court, and is therefore called the United States rule. One feature of this rule provides that, "if any payment be less than the interest, the surplus of interest must not be taken to augment the principal, but interest continues on the former principal until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied toward discharging the principal." Tell this to the pupils, and show how it would be applied in **7** if the payment made Sept. 25 had been \$10. Show how it would be applied if the September and January payments had been \$10 and \$6 respectively. Do the same with other exercises.

When partial payments are made on accounts and on notes running a short period of time, the interest is sometimes computed by the following rule: Find the amount of the principal from the time it begins to draw interest to time of settlement. From this amount subtract the sum of the payments and their interests from the time of payment to the time of settlement, and the difference will be the balance due. This rule is sometimes called the merchants' rule. The following solution of **5** is by this rule:

Principal, on interest from April 1, '92,		\$300.00
Interest to March 1, '93 (11 mo.),		16.50
Amount,		\$316.50
Payment June 1, '92,	\$50.00	
Interest to March 1, '93 (9 mo.),	2.25	
Payment Oct. 1, 92,	60.00	
Interest to March 1, '93 (5 mo.),	1.50	
Payment Dec. 1, '92,	30.00	
Interest to March 1, '93 (3 mo.),	.45	
Payment Jan. 1, '93,	40.00	
Interest to March 1, '93 (2 mo.),	.40	
Sum of payments and interests,		184.60
Balance due March 1, '93,		\$131.90

Ask the pupils to perform **6** and **7** by the above method, and compare answers with the answers obtained by the United States rule.

39 *Answers:* **2** $195.06+ $197.49+. **3** $1427.33. **4** $747.34+. **5** $248.94+. **6** Simple int. $30.14 greater.

In **1**, draw from the pupils the fact that compound interest is interest on interest. Savings banks generally allow compound interest on deposits. In **3**, interest is calculated for five separate times, the last being 8 mo.

The answers to question calling for blanks to be supplied are as follows: 1 yr. @ 4%, 1.04; 1 yr. @ 5%, 1.05; 3 yr. @ 3%, 1.092727; 3 yr. @ 5%, 1.157625; 4 yr. @ 4%, 1.169859; 5 yr. @ 3%, 1.159274; 5 yr. @ 5%, 1.276282; 6 yr. @ 3%, 1.194052; 6 yr. @ 4%, 1.265319; 7 yr. @ 3%, 1.229874; 7 yr. @ 5%, 1.4071; 8 yr. @ 4%, 1.368569; 8 yr. @ 5%, 1.477455; 9 yr. @ 3%, 1.304773; 10 yr. @ 5%, 1.628895.

40 *Answers:* **2** $1458. **3** $166.46. **4** $636 $710.16 $101 $730.20. **5** $1786.20 $1257.64. **6** $2668.73.

In the instance given in **1**, the account stands as follows, if no interest is paid until maturity of note:

Principal,		$100.00
Total simple interest (3 yr. @ 6%),		18.00
Interest on 1st annual interest (2 yr.),	$0.72	
" " 2d " " (1 yr.),	0.36	1.08
Amount due at the end of 3d year,		$119.08

The amount by compound interest for the same time and rate is $119.10, only 2 cents more than the amount by annual interest. In **4**, by the process indicated, $609.16 would be due Sept. 1, 1886. If the note were paid at this time, $101 should be deducted from $609.16. The simplest solution of this problem is to find the amount due as if no payment had been made, and then subtract $100 plus the interest of $100 from July 1, 1886, to time of settlement. **5** should be performed in the same way. Let the pupils

perform **6** in two ways, one by the United States method, and the other as if the interest at 6%, payable annually, was unpaid until maturity of note. The answer given above is obtained by the latter method.

41 *Answers:* **1** \$9600. **2** 1 mo. 1 da. 12 yr. 6 mo. **3** \$100 gain. **4** 3 yr. 11 mo. 1 da. 2 yr. 11 mo. 8 da. **5** 80%. **6** \$545.85. **7** 3 yr. 5 mo. 22 da. **8** Cash offer, \$45.45 cheaper. **9** \$64800. **10** \$1243.92. **11** First by $2\frac{8}{73}$%. **12** \$2790.62. **13** \$1115.075 \$1104.50 \$1115.36.

42 *Answers:* **1** The second. **2** \$13333$\frac{1}{3}$ \$35555$\frac{5}{9}$. **3** \$1949.85. **4** $5\frac{1}{4}$%. **5** \$15238$\frac{2}{21}$. **6** \$444.05. **7** \$5214.285. **8** \$13.50 **9** 16¢ 18¢. **10** 22$\frac{2}{9}$ yr. 11$\frac{3}{17}$ yr. **11** \$1459.44. **12** \$480000. **13** \$887.97. **14** \$307.80.

43 National Banks, before issuing any bills of their own, are obliged to put a certain portion of their capital into government bonds, which are held in trust for the banks by the Treasury Department at Washington. The bills "payable to bearer" "on demand" are printed at Washington, and cannot exceed in amount 90% of the face value of the bank's deposit in the Treasury Department. The stockholders of a bank are its owners, and may share all profits and losses as partners in proportion to the number of shares they hold. If a bank lessens its capital, or is unable from any cause to pay its debts, the stockholders are obliged to make up the deficiency not exceeding the par value of the shares which each stockholder owns. The books and papers of National banks are periodically inspected by bank examiners appointed by the government. The board of directors, besides electing the president, cashier, and, if needed, other officials, are supposed to attend to the general management of the bank.

The income of a bank is derived from the amount received in discounting notes, in collection of notes, in the interest on government bonds and other securities, and in profits of buying and selling property. The expenses are the taxes it has to pay to the

general government and to the state and local governments, and the running expenses, such as rents, salaries, etc.

The above facts, and others which may be gained from bank authorities, should be given to the pupils.

44 The method of borrowing money at a bank should be shown in a familiar and practical way, the example here given being taken as a basis. By questioning and telling, the following facts should be brought out: It is supposed that the payee goes to the bank to-day to get the note discounted. He has first to indorse the note, that is, to write his name on the back of it. By this indorsement he guarantees the payment of the note. If the officials of the bank require greater security, another person must indorse the note. The payee receives from the bank in cash \$6.15 less than \$300. This is called the avails, and is really less than the true value of the note. If, instead of presenting the note to-day, the payee should present it one month later, the bank would have to wait but 2 months before receiving \$300. It therefore discounts the interest of \$300 for 2 months and 3 days. At the end of 3 months, or within 3 days after maturity of the note, Mr. Douglas is supposed to pay \$300 to the bank. If he does not do this after due notice, a notice called a "protest" is sent by a notary public to the payee, and, if he does not pay, then to the other indorser. The indorser must pay the amount at once, so that the bank may not lose, but he can hold the maker responsible for payment afterwards.

Blanks illustrating the business of a bank can be easily obtained and shown to the pupils. The officials will be willing to give all needed information.

The following definitions may be evolved from actual examples given:

The *avails* or *proceeds* of a note is the sum that the bank pays for it after deducting the interest on the face of the note for the time before maturity.

The *face* of the note is the sum of money written in the body of the note.

The *maturity* of the note is the time at which the note is due.

Bank Discount is the allowance made to a bank for the payment of a note before it becomes due.

The *maker* of the note is the person who signs his name to the note.

The *payee* is the person to whom payment of money is promised.

The *indorser* is the person who writes his name on the back of the note.

Days of grace are days after a note is payable before it is legally due. If the last day of grace falls on Sunday or on a legal holiday, the note must be paid on the day previous. In the state of New York no grace on notes, etc., is allowed by law.

Practice varies in estimating the time of maturity and period of discount of notes. Some banks in estimating the time of maturity of a note due in 90 days regard the time as three calendar months, but more commonly such time is taken in exact days. In discounting notes some banks always reckon the time before maturity in exact number of days, others in months and days, and still others in exact number of days if the unexpired time is 60 days or less, and in months and days if for more than 60 days. The problems of this section are reckoned on the basis of months if the time given in the note is months, and of exact number of days if the time given is in days. For example, the maturity of a note dated July 15, due in 3 months, is Oct. 15 + 3 days of grace, or Oct. 18. If the note should read "90 days after date," the note would fall due Oct. 13 + 3 days, or Oct. 16. If the first of these notes is discounted at date of note or any time subsequently, the time of discount is reckoned as months and days. The discount of the second note is reckoned in exact number of days.

45 *Answers:* **1** \$493.87. **2** \$590.70. **3** \$594.75. **4** \$787.60. **5** \$444.75. **6** \$296.85. **7** \$670.65. **8** \$1267.63. **9** \$182.48. **10** \$343.67 \$346.53. **11** \$674.05. **12** Nov. 15, 1893 \$628.01 \$620.44 \$612.53. **13** \$452.92 \$455.21.

46 *Answers:* **1** \$100 \$1200. **2** \$750. **3** \$1000. **4** \$510.46. **5** \$1755.91. **6** \$561.59 \$28.57. **7** \$1012.42. **8** \$8.20 \$591.80. **9** \$607.85 \$7.85. **10** \$2000 \$1979. **11** \$6.35

$1253.65. **12** $482 $344.82. **13** $600 $597.525. **14** $5.51 $420.79. **15** $15.97.

The explanation of such problems as **1** may be: "If a note for *one* dollar will give proceeds of $.9895, it will take a note for as many dollars to give proceeds of $98.95 as $.9895 is contained times in $98.95, which is 100. *Answer:* $100."

47 The deposit slip, with the bank book, is passed to the teller, who writes in the bank book the amount of deposit. The check by which $80 is drawn out is signed by the person who wants the money, and is made payable to himself or to his order. The check given to James Smith may pass through various hands until finally it comes to the Emporia Bank. It would be well for the teacher at this point to explain the bank check and to show its usefulness to business men and others. To do this it will be necessary to show the necessity of first making a deposit of money at the bank, and to tell exactly how it may be used in paying persons who live at a distance as well as persons near the bank upon which the check is drawn. An example may be given of James Robinson living in Albany, and wishing to pay a creditor in Omaha. If he has a deposit in a bank in Albany, he has only to fill out a blank similar to the blank check represented on this page and send it to the creditor in Omaha, who carries it to a bank there for payment. If he is known, he will receive the face value of the check; if he is not known, he must be "identified" by some one before he can get the check "cashed." The banks all have dealings with one another, and therefore the check is taken by the Omaha bank and returned to Albany. If Mr. Robinson has no deposit in a bank, he may make a special deposit of a sum equal in amount to the face value of the check which he wishes to send to Omaha, or he may for a small amount buy a "cashier's check" for the desired amount, payable to the order of Mr. Robinson or to his creditor. If to the former, Mr. Robinson writes, "Pay to the order of John Brown," and signs his name. Find out from bankers the exact working of a "Clearing House," and explain the process to the pupils.

48 *Answers:* **1** $683.41. **2** $803.08.

Coöperative banks have been very extensively established, and are likely to be more common in future. For this reason, and because it will be useful for people to know their purpose and methods of operation, they should be the subject of study in school. Their nature and methods vary so much in different parts of the country that it has not been thought best to introduce in the text-book more than a statement of their general features. It would be well to get from the officers of the nearest coöperative bank its method of operation and give to the pupils problems which are likely to come to patrons of the bank, both as lender and as borrower. Describe fully its advantages, and make the problems as practical as possible.

49–50 In naming the different kinds of corporations, only those that are best known need be given, such as, banks, railroads, and large manufactories. It would be well while discussing these subjects to show the pupils the various documents that are used, such as, charter, bond with coupons, and bond certificate; also certain blanks which banks and other business houses have to fill out. Lead the pupils to see clearly the difference between bonds and stocks, bondholders and stockholders, interest and dividend. Answers to the most difficult questions given on these pages, and some others that ought to be answered, are contained in the following brief statement:

A *corporation* is an organization which has a charter to do business under the laws of the state. Its *capital* is divided into shares owned by the *stockholders*, each of whom holds a *certificate of stock* stating the number of shares owned by that person. The stockholders elect a board of directors, who attend to the general management of the corporation. The par value of the shares is the original or face value. If for any reason the shares are sold for less than their par value, they are said to sell below par, or at a discount; if for more than their par value, they are said to sell above par, or at a premium. The profits of the business are divided among the shareholders in proportion to the number of shares that each one holds. These profits are called a *dividend.*

If for any reason a corporation desires to increase its capital, it can make assessments on the stockholders, that is, oblige them to pay a certain amount for each share owned, or it can borrow money and issue promissory notes called *bonds*, which are given to persons as security for the money they lend. They are made payable at a certain time or at regular times, and bear a specified rate of interest. Attached to each bond are printed slips stating that so much interest will be paid the bearer at a certain time. These slips are called *coupons*, and can be cut off as they are needed. If the business of the corporation is good, and the interest promised is large, the bonds are likely to sell above par, or above their face value. If the interest promised is very low, or if there is danger of not paying the interest, the bonds are likely to sell below par, or at a discount.

A *stock-broker* is a person who buys and sells stocks and bonds. If this is done for other people, he charges a commission called *brokerage*. The brokerage is generally estimated upon the par value of the stocks or bonds.

51 *Answers:* **1** \$1080 \$60. **2** \$2210 \$100. **3** 25 shares \$4762.50 $4\frac{7}{38}\%$. **4** \$4400 \$300 $6\frac{9}{11}\%$. **5** \$60 on a thousand 5.28%. **6** 4 80. **7** 8. **8** Stock $\frac{40}{63}\%$ better. **9** 600 shares. **10** \$18375. **11** $7\frac{1}{2}\%$ $6\frac{1}{4}\%$ $4\frac{4}{9}\%$ $5\frac{5}{8}\%$. **12** Stock $\frac{5}{9}\%$ better Stocks $1\frac{27}{23}\%$ better. **13** \$18987.50.

52 *Answers:* **1** 80 120 $133\frac{1}{3}$ 75 70. **2** 4%. **3** 6%. **4** 25%. **6** \$1672.50. **7** \$5400. **8** \$8910. **9** \$5835. **10** \$26193.75. **11** \$10003.13. **12** \$3069.50 \$15347.50. **13** 7% bonds. **14** Bonds about 1% better. **15** \$2000.

53 Explain to the pupils the great convenience of the use of drafts or checks, and how drafts are exchanged in different parts of the country. Show that business firms and private individuals in one part of the country owe parties in another part, and that by a system of exchange these debts may be paid without sending the money. Illustrate this by the example indicated on this page. Mr. Macomber, who lives in Chicago, owes John Smith of Boston,

$480.90. He goes to Mr. Upham, a banker in Chicago, and buys of him an order, called a draft, on Thos. S. Appleton & Co., bankers of Boston, who have an account with Mr. Upham. Mr. Macomber writes on the back of the draft, "Pay to the order of John Smith," and signs his name. This he sends to Mr. Smith, who draws the money from Appleton & Co. at the given time. If the draft had read ten days after sight, it should first be presented to Appleton & Co. for acceptance, which is done by their writing the word "accepted" and signing the firm's name across the face of the note. Ten days, + 3 days of grace, after acceptance the money is payable. This is called a "time draft," to distinguish it from a "sight draft," which is payable "at sight."

If in two different parts of the country, as, for example, Boston and San Francisco, the amount of drafts drawn in each city on the other is nearly equal, the buyer of a draft in either city will have to pay little or nothing beyond the face value. But if the amount of drafts in Boston on San Francisco is far in excess of the amount drawn in San Francisco on Boston, then the buyer of a draft in Boston will have to pay a premium to pay for the risk and expense of sending money to San Francisco, and the buyer of a draft in San Francisco may be able to get it at a discount.

54 *Answers:* **1** $1015 $1004.50. **2** $1465.50. **3** $609. **4** $831.60. **5** $531.06. **6** $732.17. **7** $879.30. **8** $762.02. **9** $990.10. **10** $804.42. **11** $609.14. **12** $5940.27. **13** $8074.83. **14** $1962.17. **15** $5\frac{11}{19}\frac{9}{3}\%$.

In **1**, the time draft costs less than the sight draft, because the banker has the use of the money for 63 days, including 3 days of grace.

$1015 − $10.50, the interest of $1000 for 63 days = $1004.50.

Another method of solution is to find the cost of $1 of exchange and multiply by $1000, thus:

$1 + $.015 = $1.015 $1.015 − $.0105 = $1.0045

$1.0045 × 1000 = $1004.50.

Another solution of **9** may be as follows: \$1 of exchange can be bought for \$1.01. As many dollars of exchange can be bought for \$1000 as \$1.01 is contained times in \$1000. \$1000 ÷ \$1.01 = 990.10. *Answer*, \$990.10.

The time draft mentioned in **9** would cost .9945 of the face. \$1000 ÷ .9945 = face.

The solution of **10** is as follows: The cost of \$1 of exchange is found by subtracting \$.0155, the interest on \$1 for 93 days, from 1000 = \$9845. To this add \$.01, the premium = \$.9945. Since one dollar of exchange can be bought for \$.9945, as many dollars of exchange can be bought for \$800 as there are times \$.9945 in \$800, etc.

55 *Answers:* **2** \$488.25. **3** \$334.62. **4** £367 14*s.* 5*d.*

Show the pupils that essentially the same principle is involved in foreign exchange that has already been explained in domestic exchange. The intrinsic value of the standard coins of two countries determines the par of exchange between those countries.

Exchange is at a premium or discount when the market value is more or less than the par value. Three bills of the "same tenor and date" are sent by different mails, so that if one is lost the other may be presented.

In the bill of exchange given, Mr. Mason is the *buyer* or *remitter* who indorses the bill payable to the order of Mr. Brown. As in domestic exchange, foreign bills or drafts are made payable both on time and at sight. Time drafts are usually quoted at a less price than sight drafts.

56 *Answers:* **1** £587 15*s.* 2*d.* **2** \$1679.69. **3** \$1104. **4** 14392 francs. **5** \$2367.47. **6** 35 bonds, \$95.85 surplus. **7** \$24.50. **8** 7%. **9** \$11520. **10** \$454.78. **11** \$325.03. **12** \$84.50. **13** \$2 more for buying land. **14** \$162.49.

The note referred to in **11** is supposed to be discounted at a bank.

57 *Answers:* **1** \$212.50 8%. **2** 14$\frac{1}{2}$%. **3** \$787.60 6.09+%. **4** 7.15+% 6.09+% 9.21+% 4.55+%. **5** 18.4+%. **6** \$304.50. **7** \$1267.55. **8** \$723.84. **9** \$679.61. **10** \$406.20 \$402.21.

58 It will not be necessary for the pupils to perform many of the problems included in **1**. A few answers are given below:

(1) 1500 Atch., $1038.75. (3) 2000 C. B. & Q., $2250. (8) 2000 Or. Sh. Line 6's, $1811.33. (16) 42 B. & M., $5717.25. (24) 5 Conn. River, $1075.63. (26) 20 Mex. Cen., $126.25. (37) 235 Bos. & Mont., $4141.88. (46) 96 Pullman, $13240.

59 *Answers:* **1** Balance March 31, $58.27. **2** Balance Jan. 31, $182.21.

If preferred, the two sides of the cash account may be placed on the opposite pages of a blank book.

60 *Answers:* **1** Balance, $221.52. **3** Balance, $1049.43.

The form suggested in **4** is given below. It has the advantage over other forms in the ease with which the balance is found, and also in the greater convenience for making out monthly bills.

L. P. WALKER.

1894.				
Jan. 1	*To Balance,*		*$18*	*60*
" "	*By Cash,*		*10*	*00*
	To Balance,		*8*	*60*
" *3*	" *10 gal. K. Oil,*	*11*	*1*	*10*
" *5*	" *2 lb. Coffee,*	*32*		*64*
" "	" *3½ lb. Cheese,*	*12*		*42*
" *9*	" *3 gal. Molasses,*	*42*	*1*	*26*
" "	" *1 bbl. Flour,*		*5*	*60*
			17	*62*
" "	*By Cash,*		*15*	*00*
	To Balance,		*2*	*62*

61–62 In making the cash account, use the form already given. The other accounts may be given in the simple form given above. The poultry account should be made precisely as the personal accounts are made, debiting all items of expense incurred

on account of poultry, and crediting all items of income from the poultry.

63 *Answers:* **1** 8 mo. 18 mo. 200 mo. **2** 18 mo. 60 mo. 800 mo. **3** In 2 mo. 11 da. **4** 2 mo. from date. **5** 82½ da. **6** 3 mo. 3 da. **7** Oct. 14. **8** 2 mo. 1200 mo. 1⅛ mo.

Explain to the pupils that the average of payments is the time at which several items of debt due at different times may be paid without gain or loss to either party.

A form of written solution for **6** is as follows:

2 mo. × 420 = 840 mo.
3 mo. × 315 = 945 mo.
4 mo. × 525 = 2100 mo.
1260) 2885 (2 mo.
2520
365

$\frac{365 \times 30}{1260} = \frac{10950}{1260}$ da. $= 7\frac{11}{126}$ da.

Answer: 2 mo. 8 da.

In finding the equated time, when in the result the fraction of a day is one half or more, reckon it one day; if less than ½, disregard it.

In **9**, assuming that the two sums owed were paid at the time the first sum was due, the manufacturer would lose the use of $1200 from June 20 to July 5, or 15 days, which is equivalent to the loss of the use of $1 18000 days, or the loss of the use of $2000 9 days. Therefore, for the manufacturer not to lose anything, the $2000 should be kept by him 9 days after June 20, or until June 29.

64 *Answers:* **1** June 16. **2** Sept. 26. **3** Feb. 11.

When each of two parties is a debtor, one way of finding the equated time of paying the balance due is by computing the interest. In **4**, it might be assumed that the account given is the account of each pupil with J. R. Spaulding. The explanation may be drawn from the pupils by such questions as are given in the exercise. Afterwards the pupils may be led to explain the problem as follows:

"In this account Spaulding owes me $100, due July 1, and $50, due July 21. I owe Spaulding $100, due July 11. I wish to find

the time when he may pay the balance ($50) due me without gain or loss to either of us. We will assume July 21 as the time of payment of all the sums. If he pays me the $100 due July 1 on the 21st of July, I shall lose the use of $100 from July 1 to July 21, or 20 days. The interest of $100 for 20 days at 6% is $33\frac{1}{3}$¢. If he pays me July 21 $50, which is due July 21, I shall neither gain nor lose. If I pay him July 21 $100, which is due July 11, I shall gain the use of $100 10 days. The interest of $100 for 10 days is $16\frac{2}{3}$¢. If payment of the balance due is made July 21, my net loss is $16\frac{2}{3}$¢. That I may not gain or lose, he must pay me the $50 as many days before July 21 as it takes $50 to gain $16\frac{2}{3}$¢, or 20 days. 20 days before July 21 is July 1, when the balance should be paid."

Spaulding owes me: $100 due July 1. 50 " July 21.

I owe Spaulding: $100 due July 11.

Assumed time of payment, July 21.

Loss, int. of $100 20 da. $= .33\frac{1}{3}$. Gain, int. of $100 10 da. $= .16\frac{2}{3}$.

Net loss, $.16\frac{2}{3}$.

Interest of $50 1 da. $= .008\frac{1}{3}$. $.16\frac{2}{3} \div .008\frac{1}{3} = 20$.

July 1 — 20 da. = July 1.

By introducing the last question a new problem is made. Assume that the last bill of goods, like the others, is due in 1 mo. Let Aug. 1 be the assumed time of payment. The written solution by a good method may be as follows:

Due	Amount	Time	Interest lost	Due	Amount	Time	Interest gained
July 1	*$100*	*31 da.*	*$.516*	*July 11*	*$100*	*21 da.*	*$.35*
July 21	*50*	*11 da.*	*.091*				
Aug. 1	*100*	*0*					
	250		*.607*				
	100		*.35*				
	$150		*.257 ÷ .0025 = 10*				

Aug. 1 — 10 days = July 21.

Explanation: If \$100 due me July 1 is paid Aug. 1, I lose the interest of \$100 31 da., or \$.516. If \$50 due me July 21 is paid Aug. 1, I lose the interest of \$50 11 da., or \$.091. If I pay, July 11, \$100 that is due Aug. 1, I gain the interest of \$100 21 da., or \$.35. If all payments are made Aug. 1, my net loss is \$.257. That I may neither gain nor lose, he should pay the balance (\$150) due me as many days before Aug. 1 as the number of days it will take for \$100 to gain \$.257. Dividing by the interest of \$150 for 1 day the quotient is 10. 10 days before Aug. 1 is July 21, the equated time.

Many problems in averaging accounts may be solved best by reducing the gain or loss of \$1 for a number of days or months, and then dividing by the balance due. For example, **4** could be solved as follows:

Spaulding owes me:
\$100 due July 1.
50 " July 21.

I owe Spaulding:
\$100 due July 11.

If settlement is made July 21,

I lose the use of \$100 for 20 da. = the use of \$1 2000 da.

I gain the use of \$100 for 10 da. = the use of \$1 1000 da.

Net loss of use of \$1 for 1000 da.
" " " " " \$50 " 20 da.
July 21 — 20 da. = July 1.

65 *Answers:* **1** June 15. **2** Nov. 18. **3** Balance, \$260, due April 1. **4** Oct. 27.

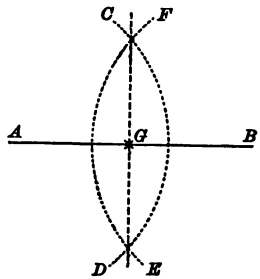
These problems are essentially the same as the problem solved above. The earliest date instead of the latest may be taken as the assumed time of payment if thought best.

66 *Answers:* **1** June 4, 1893. **2** Balance due July 16, 1894. **3** June 13, 1894. **4** March 2, 1895 \$891.09. **5** May 19, 1879 Balance due, \$44.74.

67 A horizontal line is a line having the direction of a line in the surface of still water. A vertical line is one that has the

direction of a plumb line. These lines need be drawn only approximately in the right direction. Parallel lines may be drawn by the aid of a square or ruler, care being taken to draw them so that they shall be the same distance apart throughout their length. Other ways of drawing parallel lines will be shown later.

A line may be divided into 2 equal parts by means of a measure. Another way is to divide it with the aid of compasses, as follows: With A as a centre, draw the arc CD, and with B as a centre, draw the arc EF, cutting the arc CD. Unite the points of intersection of the two arcs, and the dividing line divides the line AB at G into 2 equal parts. Each of the two parts can be divided in the same way into 2 equal parts, thus dividing the original line into 4 equal parts.



68 Most of these exercises are a review of previous work.

69 The conclusion in **1**, after several similar experiments, is, that the sum of two sides of a triangle is greater than the third side. The two ways of finding by experiment the sum of the angles of a triangle are: (1) Cutting off the corners of a paper triangle and measuring, and (2) Measuring by means of a protractor.

Several conclusions may be made from the measurements made in **17**; such as: The two diagonals of a parallelogram divide each other into two equal parts. In a square the diagonals are equal. In a square the diagonals are perpendicular to each other. In a rhombus the diagonals are perpendicular to each other.

70 *Answers:* **1** \$80. **2** 129600 sq. ft. **3** $\frac{125}{968}$ A. **5** $11\frac{7}{363}$ sq. rd. **6** $7\frac{1}{32}$ A. **9** 2244 sq. ft. **10** 21600 sq. ft. **11** 1 T. 1966 lb. **12** 176.59+ ft. **13** $85\frac{1}{3}$ rd. **14** 19 planks \$6.08.

In **9**, the walk is supposed to be on the border of the garden.

71 *Answers:* **1** $23\frac{1}{3}$ yd. **2** \$3.30. **3** $20\frac{1}{18}$ sq. ft. **4** 869.5 sq. ft. **5** 7855.8 sq. ft. \$116.26. **6** \$46.78 \$3.92 (full allowance for openings) \$24.75 $460\frac{5}{12}$ sq. ft. **8** $118\frac{2}{11}$ sq. rd. **9** 140 ft.

For a good practical method of finding the number of rolls of paper required for a room, see page 119 of the Manual.

72 *Answers:* **1** 20000 sq. ft. $3443.53. **6** 2 A. 4 sq. rd. **7** 1 A. 126.2+ sq. rd. **11** 435.6 ft.

The land taken by the railroad company in **1** is a parallelogram whose base is *ef* and whose altitude is equal to the distance *bd*.

For solution of **2**, see Book IV., page 85.

The dotted line called for in **8** is perpendicular to the base, since the altitude is the perpendicular distance from the vertex to the base. Let the statements be as concise as possible.

73 *Answers:* **1** $56\frac{10}{11}$ sq. rd. 1936 yd. $454\frac{53}{181}$ ft. $4826\frac{5}{8}$ sq. ft. 16 A. $3\frac{1}{3}$ sq. ch. 45 A. $8\frac{17}{32}$ sq. ch. $15\frac{5}{7}$ rd. **2** 1200 sq. ft. **3** $1212\frac{1}{2}$ sq. ft. **4** $1732\frac{1}{2}$ sq. ft. **5** $6383\frac{1}{2}$ sq. ft. **6** $1758\frac{3}{4}$ sq. ft. **7** 900 sq. ft. **8** 920 sq. rd.

The line required to be drawn 60 ft. long in **7** is supposed to be horizontal.

74 *Answers:* **2** $8\frac{1}{3}$ sq. ft. **3** 22400 sq. ft. 46200 sq. ft. **5** 2 inches. **6** Area of triangle, 2400 sq. ft.; of trapezoid, 2100 sq. ft. **7** 1450 sq. ft.

The cut on page 12, Book VI., will suggest a solution for **1**.

4 may be solved by construction, after transforming a trapezoid into an equivalent rectangle; also from the formula $A = \frac{B+b}{2} \times h$.

75 *Answers:* **1** 6 sq. ft. **2** 12600 sq. ft. **11** 36.34 sq. ft. **14** 25.1328 ft. **15** 4.77+ ft. **16** 314.16 sq. ft. **17** 9.58+ A. **18** $4.36.

Suggestive helps to the solution of these exercises will be found on pages 14–18 of Book VI.

Use protractor or compasses in marking off arcs of various degrees.

A *regular polygon* is a polygon having equal sides and equal angles. In drawing regular polygons, draw circles first and mark off equal arcs of the desired length.

Let the pupils divide paper circles into triangles before drawing if they have not already done so (see page 18, Book VI.).

76 *Answers:* **1** 502.656 sq. ft. **3** 22 ft. 138.2304 ft. 175.9296 ft. 932.48 sq. ft. **4** $41.30. **5** 492 A. 145.4+ sq. rd.

In **1**, it is supposed that the cow is tethered in such a way as to enable her to graze at a distance exactly forty feet from the post.

Somewhat similar work with prisms is called for on page 19, Book VI. For suggestions in teaching these forms, see Manual, page 120. The following statements may be drawn from the pupils:

The lateral faces of all right prisms are in the form of rectangles. The bases of a triangular prism are triangles. The bases of quadrangular prisms are quadrilaterals; etc. A *pentagonal prism* is a prism having pentagons for its bases. A *right prism* is a prism whose lateral edges are perpendicular to the bases. An *oblique prism* is a prism whose lateral edges are inclined to the bases.

The drawing on page 64, Book V., will suggest the kind of work called for in **8**.

77 *Answers:* **1** 245 cu. ft. **2** 1152 cu. in. **3** $57\frac{27}{35}$ bbl. **8** 20.5632 gal. **9** 8 ft. 6.1241— in. 21 ft. 9.4379 in. **10** 79.5872 sq. ft.

The *altitude of a pyramid* is the perpendicular distance from the vertex to the base. The *slant height of a regular pyramid* is the perpendicular distance from the vertex to any side of the base. (First show that a regular pyramid is a pyramid which has a regular figure for the base, and the vertex directly above the center of the base. All pyramids referred to in this book are regular pyramids.) Teach and ask the pupils to describe triangular, quadrangular, pentagonal, hexagonal, and octagonal pyramids.

For suggestions in answering **5** and **6**, see page 20, Book VI.

The *altitude of a cone* is the perpendicular distance from the vertex to the base. The *slant height of a cone* is the distance from the vertex to any point in the circumference of the base. (All cones referred to in this book are right circular cones. A *right*

circular cone is a volume that may be generated by the revolution of a right-angled triangle about one of its legs as an axis.)

78 *Answers:* **3** 128 cu. ft. **4** 93537284 cu. ft. **5** 386.34 sq. in. **6** $6\frac{3}{4}$ in. **7** 15.919 cu. ft. **8** 1.005$\frac{1}{5}\frac{7}{5}$ T. **9** 141.372 sq. ft.

Paper will have to be pasted over the joined edges of these models, that they may hold the sand. Let the pupil measure or weigh the sand as accurately as possible. The rules which they should discover are as follows: The volume of any pyramid is equal to one third of the volume of a prism having the same base and altitude. The volume of a cone is equal to one third the volume of a cylinder having the same base and altitude. From these rules may be derived the following: The volume of a pyramid or of a cone is equal to $\frac{1}{3}$ of the area of the base $\times$ the altitude. The convex surface of a pyramid is the sum of the lateral faces. The convex surface of a cone is its curved surface. Lead the pupils to give the rule for finding the area of the convex surface of pyramids and cones. The rule is: Multiply the perimeter of the base of the pyramid or cone by one half the slant height.

79 *Answers:* **4** *a* 414.6912 sq. ft.; *b* 779.1168 sq. ft.; *c* 1381.32225 sq. ft. **5** *a* 175.92 cu. in.; *b* 2865.13 cu. ft. **8** 1256.64 sq. in. 2827.44 sq. in. 5 sq. ft. 13.3+ sq. in. 534.83+ sq. in. **9** 135.0899 cu. in. 4.1888 cu. ft. 904.7808 cu. in. **10** $1884.96. **11** 512 bullets 110609 bullets.

The *frustum of a pyramid* is the portion of a pyramid included between the base and a plane made by cutting the pyramid through the lateral faces, parallel to the base.

The slant height of the frustum of a pyramid or of a cone is the perpendicular distance between the sides of the upper and lower bases. The total surface of a frustum is found by adding the surface of the bases to the lateral surface. The lateral surface is found by multiplying half the sum of the perimeters of the bases by the slant height.

The volume of the frustum of a pyramid or cone is equal to the difference between the original pyramid or cone and the pyramid or cone cut off. If the height of the original cone is not given, the volume of the frustum is found by the following rule: To the sum of the squares of the diameters of the two bases add the product of the two diameters. Multiply this sum by the product of $\frac{1}{3}$ of the height and .7854.

With an orange or wooden ball teach the following definitions:

A *sphere* is a solid bounded by a curved surface, every point of which is equally distant from a point within called the centre.

The *diameter of a sphere* is a straight line passing through the centre and having its extremities in the surface.

A *great circle* of a sphere is a section made by cutting the sphere through the centre.

A good method of finding the surface of a sphere is to compare it with the surface of a right cylinder whose height and diameter of base are exactly equal to the diameter of sphere. By wrapping the sphere and cylinder with narrow waxed tape, and, after unwrapping them, comparing the amounts of tape, it will be observed that the areas of the surface of the sphere and the lateral surface of the cylinder are alike. Since the lateral surface of a cylinder is found by multiplying the circumference of the base by the altitude, it will be seen that the surface of a sphere is found by multiplying the circumference by the diameter.

To find the volume of a sphere, compare the contents of the sphere with the contents of the enveloping cylinder. This may be done by making a ball which will exactly fit a cylinder. Fill the cylinder with water and immerse the ball. Compare the depth of water before the ball was immersed with the depth of water after the ball is taken out, and it will be found that two thirds of the water has been displaced. Therefore, the volume of a sphere is equal to two thirds of the volume of a cylinder whose diameter of base and height are equal to the diameter of sphere. From this fact may be derived the rule: Multiply the cube of the diameter by .5236; or, by conceiving a series of pyramids formed from the

sphere, having their vertex at the center, it will be seen that the volume of the pyramids composing the sphere must equal the surface of their bases multiplied by $\frac{1}{3}$ of their radius; hence the rule: To find the volume of a sphere, multiply the surface by $\frac{1}{3}$ of the radius.

80 *Answers:* **3** 90° 60° 30°. **5** $69\frac{1}{6}$ mi. **6** 12430.8 mi. **7** 598.1 mi. 2392.4 mi. 10765.8 mi. 21531.6 mi. **8** About $69\frac{1}{6}$ mi. **9** 47° $3250\frac{5}{6}$ mi. **10** 43° $2974\frac{1}{6}$ mi.

If necessary, review previous work in circles and degrees, pages 14, 15, Book VI.

81 *Answers:* **2** 240′ 14400″. **3** 640′. **4** 15600″. **5** 495′ 405′ 10′ 950′. **6** 4200″ 15630″ 22320″ 900″ 10″. **7** 600″ 12″ 7650″. **8** 14800″ 192″ 2340″ 21″. **9** $\frac{37}{360}°$ $\frac{1}{80}°$ $\frac{11}{180}°$. **10** $10\frac{5}{6}°$ $.06\frac{1}{3}°$. **11** $.75\frac{5}{18}°$ $.01\frac{1}{4}°$. **12** 10° 50′ 40″. **13** 36° 31′ 30″. **14** 9° 34′ 44″. **19** 23° 10′. **20** 12° 20′. **21** 89° 55′ 22″.

82 *Answers:* **1** 6°. **2** $11\frac{2}{3}$ da. **3** $73\frac{2}{45}$ h. **4** 2° 31′. **5** 7′ $59\frac{8}{33}\frac{3}{3}$″. **9** 360° 15° 15′. **10** 3° 45°. **13** 40 min. 26 min. 1 h. 41 min. 20 sec. **14** Slower 4 min. **15** 11.03 P.M. 1.57 P.M. **16** 132° 10′ E.

83 *Answers:* **2** N.Y., 7 h. 3 min. 44 sec. A.M. Chi., 6 h. 9 min. 28+ sec. A.M. N.O., 5 h. 57 min. 23+ sec. A.M. London, 11 h. 59 min. 37 sec. A.M. Paris, 12 h. 9 min. 33+ sec. P.M. Boston, 7 h. 15 min. 46 sec. A.M. Wash., 6 h. 51 min. 57 sec. A.M. Rome, 12 h. 49 min. 4+ sec. P.M. Berlin, 12 h. 53 min. 34+ sec. P.M. San F., 3 h. 50 min. 26+ sec. A.M. Cal., 5 h. 53 min. 20 sec. P.M. St. L., 5 h. 59 min. A.M. **3** Rome, 9 h. 57 min. 50+ sec. P.M. Paris, 9 h. 17 min. 22+ sec. P.M. Cal., 3 h. 1 min. 22 sec. A.M. **5** 15°. **6** 21° 15′. **7** 5° East. **8** 9 h. 3 min. $8\frac{4}{15}$ sec. **9** 5 h. 50 min. $8\frac{3}{5}$ sec. **10** 20°. **11** 93° 20′ West. **12** 13° 22′ 6″.

84 *Answers:* **2** Ind., 12 o'clock. Pitts., 1 o'clock P.M. Denver, 11 o'clock A.M. San. F., 10 o'clock A.M. N. O., 12 o'clock. Boston, 1 o'clock P.M. Omaha, 12 o'clock. **3** San Jose, 1 o'clock P.M.

Portland, 4 o'clock P.M. Springfield, 3 o'clock P.M. **4** Charleston, 10 h. 15 min. A.M. Sac., 7 h. 15 min. A.M. **5** Cin., 22 min. 19 sec. Wil., 11 min. 30 sec. St. P., 12 min. 20 sec. **7** 1 min. 16 sec. past 4 28 min. 20 sec. past 1.

85 The relation or ratio of 6 blocks to 2 blocks is found by dividing 6 blocks by 2 blocks = 3. The ratio is expressed thus: 6 blocks : 2 blocks. The antecedent of this ratio is 6 blocks, and the consequent is 2 blocks.

Lead the pupils to make the following statements from examples called for in **11**:

If both terms of a ratio are multiplied by the same number, the terms are larger, but the ratio remains unchanged.

If both terms of a ratio are divided by the same number, the terms are smaller, but the ratio remains unchanged.

86 A proportion is the equality of ratios. The first and fourth terms of a proportion are called the extremes. The second and third terms are called the means. From observation the pupils will discover and state the conclusion that the product of the fourth term by the number of units in the first term equals the product of the third term by the number of units in the second term; or, more briefly, the product of the extremes equals the product of the means. From this fact it will appear that the product of the extremes divided by a mean equals the other mean; and that the product of the means divided by an extreme equals the other extreme.

87 *Answers:* **4** *a* \$8; *b* \$90; *c* \$.25; *d* \$.52½; *e* 12 lb.; *f* 16⅔ lb.; *g* \$625; *h* \$2.13½; *i* \$.47½; *j* \$18.75. **5** *a* \$.10 1/16; *b* \$.13⅓; *c* \$1⅛; *d* \$1.50; *e* \$.15; *f* \$200; *g* 3 lb.; *h* 6 ex.; *i* \$.07½; *j* \$.60. **6** 12¢. **7** \$4.80. **8** \$.80. **9** 3 doz. **10** \$.50.

The last five problems on this page and all the problems on the five following pages are intended to be performed by analysis and by proportion. The solution by analysis should be either oral or on a line. The solution by proportion may be as follows (**6**): The ratio of 2 apples to 6 apples must be the same as the cost of

2 apples to the cost of 6 apples. It is required to find the cost of 6 apples; therefore we make 4 cents, the cost of 2 apples, the 3d term. Since the greater the quantity the greater the cost, we make 6 the 2d term and 2 the 1st term. The proportion is $2 : 6 = 4¢ : x$. Multiplying the 4 by 6 and dividing the product by 5, we have for the fourth term 12. *Answer*, 12¢.

88 *Answers:* **1** 20 apples. **2** \$80. **3** 25 bu. **4** \$14.25. **5** $66\frac{2}{3}$ mi. **6** \$22.40. **7** \$1120. **8** $145\frac{5}{6}$ A. **9** 1488 bu. **10** 450 rd. **11** 2400 T. **12** $53\frac{1}{3}$ da. **13** $4\frac{4}{5}$ da. **14** \$13230. **15** 20 ft. **16** $111\frac{2}{3}$. **17** 480 men. **18** $29\frac{1}{11}$ mi. **19** $103\frac{3}{11}$ ft.

The solution on a line may be as follows (**13**):

$$\frac{\overset{2}{\cancel{8}}\text{ da.} \times 12}{\underset{5}{\cancel{20}}} = \frac{24}{5} = 4\frac{4}{5}\text{ da.}$$

If 12 men mow the meadow in 8 days, it will take 1 man 12 times as many days to mow it as it takes 12 men, and 20 men will mow it in one twentieth as many days as it takes 1 man. Multiplying and dividing, we have an answer of $4\frac{4}{5}$ days.

89 *Answers:* **1** $461\frac{7}{13}$ mi. **2** $\$2346\frac{2}{3}$. **3** $13\frac{7}{8}$ yd. **4** $12\frac{1}{2}¢$. **5** \$1105.92. **6** 66000 times. **7** 277 da. 18 h. 40 min. **8** 258000 men. **9** $10\frac{1}{11}$ rolls. **10** $18\frac{3}{4}$ lb. **11** 1152 mi. **12** $16\frac{2}{3}$ h. **13** \$677.00+. **14** \$32000. **15** $444\frac{1}{2}$ lb. **16** 1 lb. 14+ oz.

90 *Answers:* **1** 4¢ and 8¢. **2** 30 yr. and 10 yr. **3** $\frac{3}{10}$. **4** A, \$8; B, \$16. **5** A, \$12; B, \$10; C, \$8. **6** 11¢, 22¢, 33¢. **7** $\$186\frac{2}{3}$, \$280, $\$373\frac{1}{3}$. **8** A, \$1008; B, \$504; C, \$168. **9** 12, 8, 4. **10** 315 and 525. **11** 18 rd. and 24 rd. **12** Each daughter, \$8000; each son, \$6000. **13** 18 rolls. **14** $3389\frac{4}{5}\frac{8}{5}$ lb. **15** $6\frac{2}{3}$ A. **16** 4352 lb. **17** 125 lb.

In such problems as **6**, the whole number will be the sum of the parts 1, 2, 3, and may be represented by 6. The proportions will be, $6 : 1 = 66 : 11$; $6 : 2 = 66 : 22$; $6 : 3 = 66 : 33$. By analysis the solution is, $66 = \frac{6}{6}$; $\frac{1}{6} = 11$; $\frac{2}{6} = 22$; $\frac{3}{6} = 33$.

In **9**, reduce the fractions to a common denominator and use the numerators, as in **6**.

91 *Answers:* **1** A, \$400 B, \$800. **2** A, \$4800 B, \$2880 C, \$4320. **3** A, \$635$\frac{5}{17}$ B, \$264$\frac{12}{17}$. **4** A, \$1800 B, \$2700. **5** Hall, \$888 Reed, \$592. **6** A, \$3777$\frac{1}{3}$ B, \$5666$\frac{2}{3}$ C, \$7555$\frac{1}{3}$. **7** A, \$626$\frac{2}{23}$ B, \$1173$\frac{21}{23}$. **8** A, \$64$\frac{8}{13}$ B, \$43$\frac{1}{13}$ C, \$32$\frac{4}{13}$. **9** A, \$49$\frac{3}{13}$ B, \$30$\frac{10}{13}$. **10** A, \$15600 B, \$20400 C, \$24000. **11** A, \$3840 B, \$4800 C, 3360.

In **7** to **11**, make the conditions uniform before the proportion is made. A's share of the work done was equal to the service of 192 men 1 day, and B's share was 360 men 1 day. A's share of the contract money should be, therefore, $\frac{192}{552}$ of \$1800.

92 *Answers:* **1** 5$\frac{1}{8}$ da. **2** 2 gal. **3** 9$\frac{1}{8}$ weeks. **4** 320 mi. 21$\frac{3}{7}$ da. **5** 5 mo. **6** \$240. **7** 11$\frac{1}{4}$ da. **8** 370$\frac{10}{27}$ rd. **9** 272$\frac{112}{189}$ 176$\frac{1}{24}$. **10** 5$\frac{1}{8}$ rd. **11** 8¢ 2¢ 6 of 40¢ and 4 of 50¢. **12** 3 of 6¢ and 1 of 8¢ 3 of 6¢ and 5 of 8¢.

These problems should be performed by analysis on a line, rather than by compound proportion. The solution of **8** may be as follows:

$$\frac{\overset{40}{\cancel{80}}\text{ rd.} \times \overset{5}{\cancel{40}} \times 50}{\underset{9}{\cancel{18}} \times \underset{3}{\cancel{24}}} = 37\tfrac{1}{17}\text{ rd.}$$

My answer is to be in rods, therefore 80 rods is the number to work upon. 1 man will build $\frac{1}{18}$ as many rods as 18 men, found by dividing by 18; 40 men will build 40 times as many rods as 1 man, found by multiplying by 40. If they can build so many rods in 24 days, in 1 day they will build $\frac{1}{24}$ as many rods, found by dividing by 24, and in 50 days they will build 50 times as many rods as they build in 1 day, found by multiplying by 50. Simplifying by cancellation we have 37$\frac{1}{17}$ rd.

93 Nearly all of these problems should be performed orally.

94 *Answers:* **7** 1988515.54 T. **8** 31169400000 lb. 15584700 T. **9** \$31684702.80. **10** W., 46.66% Cl., 56.89% N.Y., 44.06%

Bos., 72.13% L., 39.34% Ph., 62.71% Bal., 25% S. L., 41.37% Br., 62.71% Cin., 55.17% Pitts., 46.77% Ch., 47.36%.

95 *Answers:* **1** $25\frac{10}{797}$ ft. **2** $\frac{1}{25}$%. **3** 36 $19\frac{1}{2}$ 30. **4** $66\frac{2}{3}$% $33\frac{1}{3}$% 400%. **5** 5 3 2000 .002. **6** .08 .0003 .00001 2400. **7** 200 bolts 4000 bolts 21100 bolts $3 $3.46 $.58. **8** $24.50 $58.19 $19.69. **9** $24.75. **10** $50.63. **11** $1513.51. **12** 70%. **13** $10200. **14** 95 shares ($235 left). **15** 34.6+%. **16** 52 shares ($80 left). **17** $88500. **18** $360. **19** $8\frac{3}{4}$%.

96 *Answers:* **1** 116 bales. **2** $3037.50. **3** $7507.58. **4** $3461538.46. **5** $25.35. **6** $69.23. **7** 42 shares. **8** 28+%. **9** $7087.50. **10** $10675 $97.50 and $154.32. **11** $10008. **12** $40099.75.

97 *Answers:* **1** $1500. **2** $362.09. **3** $7200000. **4** 3 h. 44 m. P.M. **5** $301.50. **6** $807.50. **7** $3\frac{1}{2}$%. **8** $4473.79 $192 $134.21. **9** $111.23. **10** 1 yr. 21 da. **11** $987.65. **12** $100.

98 *Answers:* **1** $393.89. **2** $685.94. **3** $4125.72+. **4** $4800. **5** 80%. **6** $925. **7** .00948. **8** $1550 $1475 $1400 $1325. **9** $6744.36. **10** $112.31+ $75 5.3+%.

99 *Answers:* **1** $8.22. **2** $26.44. **3** $110.10. **4** 5. **5** $496.13. **6** $60. **7** $106.55\frac{5}{14}$. **8** $385.71\frac{3}{7}$. **9** 960 stones. **10** $2\frac{8}{11}$ da. **11** $65\frac{1}{3}$ yd. $122.50. **12** 380 sq. mi. **13** $494\frac{8}{33}$ sec. A little more than $\frac{1}{8}$ of a second. **14** $87.22.

100 *Answers:* **1** $432.56. **2** 5616 tiles 2990 tiles. **3** $3.62\frac{1}{2}$. **4** $1441.19. **5** $1438.25. **6** 76¢. **7** $54\frac{6}{11}$ wk. **8** $18.13. **9** (*a*) $2450.25; (*b*) $20\frac{7}{88}$ cu. yd.; (*c*) $167.24+; (*d*) $36.16; (*e*) $58.58; (*f*) 2.019+ bu. **10** 2538.0864 gal. $45.24.

101 *Answers:* **1** $6\frac{13}{18}$ cu. yd. **2** 20 yd. **3** 72000 cu. ft. 120 ft. **4** 25 42.24 cm. **5** 2.891+m .18 mm. **6** 1.8° F. $\frac{8}{9}$° C. $5\frac{5}{9}$° C. 25° C. 68° F. 95° F. **7** $37\frac{7}{9}$° C. **8** 20° C. **9** 38400 cu. ft. 512000 cu. ft. **10** .655+ gal. **11** 8.6+ qt.

102 *Answers:* **1** 548 ft. **2** 12.04+ sec. **3** 1.27+ sec. **4** 23209140800000000 miles. **5** 10 lb. **6** 20 lb. **7** 3400 lb. 7000 lb. **8** $2\frac{1}{2}$ and 1. **9** $5\frac{1}{3}$ lb. **10** $83\frac{1}{3}$ lb. **11** $1\frac{1}{3}$ ft. from the end.

SECTION X.

NOTES FOR BOOK NUMBER EIGHT.

Unless the principles and processes of the various subjects of Arithmetic are familiar, it will be necessary to give some preliminary reviews before the pupils are able to give the definitions and rules called for in this book. The exercises themselves suggest some desirable kinds of review work, as well as some forms of definitions which may be made. For suggestive hints as to methods of teaching definitions and rules see Manual, pages 139, 140.

Some portions of the section on Geometry may seem too difficult for pupils of Grammar grades, but if the geometrical exercises of previous books have been fairly well performed, pupils of the highest grades should be able to take all the inventional and constructive work, much of which is a review, and the easier portions of the demonstrative work. The most difficult exercises might be given to such pupils only as show a special aptitude for the study of geometry. In this, as in other subjects, the aim should be to give work that shall be sufficient, both in amount and kind, to fully tax the powers of every pupil.

1 In **4**, lead the pupils to discover the fact that the decimal system of numbering depends upon the principle that ten units of one order equal one unit of the next higher order. Other systems should be illustrated by examples. In the duodecimal system, for example, the pupils should be led to see that $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$, can be expressed by a single figure; thus, .6, .4, .3, .2; and that $\frac{1}{8}$ and $\frac{1}{9}$ can be expressed by two figures; thus, .18 and .16.

The origin of the words expressing numbers to twelve may be found in an unabridged dictionary. Other names are derivatives, and their origin will be plainly seen.

The definitions called for on this page are as follows:

A *unit* is anything which is considered as one.

An *integral unit* is one or a collection of ones regarded as an undivided whole.

A fractional unit is a part of one regarded as an undivided whole.

A *number* is a unit or a collection of units. An *integral number* is an integral unit or a collection of integral units. A *fractional number* is a fractional unit or a collection of fractional units. Number is a quality of objects which answers the question, "how many," and arises from distinguishing one from more than one.

Arithmetic is that knowledge which has for its object the expression, the operations, and the relations of numbers.

The *sum* of two or more numbers is their united value.

Addition is the process of finding the sum of two or more numbers.

Subtraction is the process of taking away a part of a number to find how many units are left. The *minuend* is the number separated. The *subtrahend* is the number taken away. The *remainder* is the part left.

Multiplication is the process of finding the united value of two or more equal numbers. The *multiplicand* is the number multiplied. The *multiplier* is the number which shows how many times the multiplicand is taken. The *product* is the result obtained by multiplication. The *factors* of a number are the numbers which, multiplied together, produce that number.

Division is the process of finding one of the equal parts of a number, or of finding how many times one number is contained in another. The *dividend* is the number divided. The *divisor* is the number by which we divide. The *quotient* is the result obtained by division.

An *odd number* is a number which cannot be separated into two equal integral parts. An *even number* is a number which can be separated into two equal integral parts.

A *prime number* is a number whose only integral factors are itself or one. A *composite number* is a number which is composed of other integral factors besides itself or one.

A *prime factor* is a factor which is a prime number.

A *multiple* of a number is any whole number of times a number. A *common multiple* of two or more numbers is a number which is a multiple of each of the numbers. The *least common multiple* of two or more numbers is the least number which is a multiple of the numbers.

A *common divisor* or *common factor* of two or more numbers is a factor which belongs to each of them. The *greatest common divisor* or factor of two or more numbers is the greatest factor which belongs to each of them.

The *denominator* of a fraction is the number of equal parts into which the unit is divided. The *numerator* is the number of equal parts taken.

To change fractions to equivalent fractions having the least common denominator: Divide the least common multiple of the denominators by the denominator of each fraction and multiply both terms of the fraction by the quotient.

To add fractions: Change the fractions to equivalent fractions having a common denominator; add the numerators, and write the sum over the common denominator.

2 To subtract fractions: Change the fractions to equivalent fractions having a common denominator, and write the difference of the numerators over the common denominator.

To multiply a fraction by an integer: Multiply the numerator of the fraction by the integer and place the product over the denominator; or, divide the denominator of the fraction by the integer and place the quotient under the numerator.

To find the fractional part of a fraction: Write the product of the numerators over the product of the denominators.

To divide an integer or fraction by a fraction: Change the numbers to equivalent fractions having a common denominator, and divide as in whole numbers; or, invert the divisor and proceed as in multiplication.

The surveyor's chain, often called Gunter's chain, has largely gone out of use, having been replaced by the steel ribbon, commonly 100 feet long, with the principal divisions and markings

at the foot points, with other divisions at tenths and hundredths of feet.

Other points called for may be found in the Tables given at the end of Book VII. or in the Appendix of Book VIII.

3 The standard unit of dry measure is the bushel, which contains 2150.42 cu. in. This bushel is sometimes called the Winchester bushel, which was the standard bushel of England before the imperial bushel was adopted. The imperial bushel contains 2218.192 cu. in., or 80 lb. of distilled water. The standard unit of liquid measure is the gallon, which contains 231 cu. in. A chaldron, sometimes used in measuring charcoal, contains 36 bu. Barrels, hogsheads, tierces, and pipes vary in capacity. A stone, sometimes used in measuring iron and lead, is 14 lb.

The meter is the standard unit from which all metric measures are derived. It is intended to be the ten-millionth part of the distance on a meridian from the equator to the pole. Lead the pupils to see the connection of all parts of the metric system — a cubic centimeter of water (at 39° F.) weighing 1 gram, and a cubic decimeter of water measuring 1 liter.

A *solar year* is the average time it takes the earth to make a complete revolution around the sun, or about 365 da. 5 h. 48 min. 49 sec. This, it will be seen, is about 365$\frac{1}{4}$ days, and therefore one extra day is counted every fourth year. But by this reckoning we should gain nearly a day in one hundred years, and therefore the centennial years are commonly counted as ordinary years. If all the centennial years were thus counted, we should lose nearly a day in four hundred years; therefore, only the centennial years divisible by 400 are counted as leap years. It would be well in this connection to tell the pupils how and when this new style of reckoning the years was adopted.

The *lunar month* is the time between two new moons, and is about 29 da. 12 h. 44 min. long.

All other measures and values referred to on this page may be found in the Appendix.

4 The *percentage* is a number which is a certain number of hundredths of another number.

The *base* is the number of which a number of hundredths is taken to find the percentage.

The *rate* is the number of hundredths which the percentage is of the base.

The *amount* is the sum of the base and percentage.

The *remainder* is the difference between the base and percentage.

Lead the pupils to find these terms in Profit and Loss, Insurance, Duties, Interest, etc., and to give examples as required in **2** to **4**.

Other points referred to on this page are explained in Section IX. of the Manual.

5 In **2**, the formula is $B = P \div R$. In **3**, $B = A \div (1 + R)$. While it is not advisable generally for pupils to perform problems in percentage by formulas, it is good practice occasionally to perform them in that way, especially when the formulas are made by the pupils.

6 Numbers may be added in pairs instead of one at a time; thus, in adding 28, 36, 43, 84, 68, 74, either in a column or in a line, the process might be 12, 19, 33; 10, 24, 31, 33. *Answer*, 333. If there are many numbers to be added they might be divided into sections and the separate sums added. Two columns may be added at once by first adding the column of the higher denomination; for example, in adding the numbers of this column the addition might be made as follows: 45, 105, 109, 129, 134, 164, 171, 211, 219.

48
37
25
64
45

In **6**, let the pupils see that multiplication by the aliquot parts of 10, 100, and 1000 may be made by annexing one, two, or three ciphers and dividing. To multiply by any number between 91 and 99, multiply by 100 and subtract. The same process may be followed in multiplying by 99, 999, etc.

In dividing by the aliquot parts of 100, first divide by 100, and then multiply.

To add fractions having 1 for the numerator, place the sum of the denominators over their product. To multiply any number containing $\frac{1}{2}$, by itself. Multiply the whole numbers, and to the

product add $\frac{1}{2}$ of the sum of the whole number and $\frac{1}{4}$; for example: $16\frac{1}{2} \times 16\frac{1}{4} = 16 \times 16 = 256 + (\frac{1}{2}$ of $32) + \frac{1}{4} = 272\frac{1}{4}$. From this process lead the pupils to make a general rule for the multiplication of mixed numbers in which the fractions are alike.

To multiply mixed numbers when the whole numbers are alike: Multiply the whole numbers, and to the product add that part of one of them expressed by the sum of the fractions, and to this sum add the product of the fractions; thus: $16\frac{3}{8} \times 16\frac{1}{4} = 16 \times 16 = 256$; $\frac{5}{8}$ of $16 = 10$; $\frac{3}{8} \times \frac{1}{4} = \frac{3}{32}$. $256 + 10 + \frac{3}{32} = 266\frac{3}{32}$, *Ans.*

7 These exercises are intended to be performed with the fewest possible figures by the shortest method. Let the pupils practice upon them until facility is acquired.

8 *Answers:* **1** 2 yr. 2 mo. 12 da. **2** \$62.50. **3** \$90 more. **4** 6487.15+ bu. **5** $7\frac{9}{13}\%$. **6** \$300. **7** \$4565.34 $6\frac{561834}{3728361}$, or about $6\frac{1}{8}\%$. **8** Atch. the better by $\frac{11}{183}\%$.

9 *Answers:* **1** \$8666$\frac{2}{3}$. **2** 30%. **3** 200% 20 men 7 men. **4** 9.45 cu. in. **5** $59\frac{44}{77}$ qt. $51\frac{3}{7}$ qt. **6** $538\frac{48}{77}$ gal. **7** $10\frac{34}{75}$ in. **8** $2\frac{181}{184}$ ft. **9** $8\frac{1}{8}$ ft. **10** $820\frac{1}{8}$ T. **11** $4712\frac{29}{144}$ pwt. **12** $66\frac{2}{3}$ lb. 75 lb. 80 lb. $44\frac{4}{9}$ lb.

10 *Answers:* **1**

Languages.	Number of Persons Spoken by.		Percentage of Increase in Eighty-nine Years.	Percentage of the Whole.	
	1801.	1890.		1801.	1890.
English	20520000	111100000	441.4	12.7	27.7
French	31450000	51200000	62.7	19.4	12.7
German	30320000	75200000	115.03	18.7	18.7
Italian	15070000	33400000	180.91	9.3	8.3
Spanish	26190000	42800000	157.6	16.2	10.7
Portuguese	7480000	13000000	171.7	4.7	3.2
Russian	30770000	75000000	71.8	19.0	18.7
Total	161800000	401700000	248.26	100.0	100.0

2 \$2750. **3** 36. **4** June 21, 5 h. 23 min. 30 sec. P.M. **5** \$24.53. **6** About 12 acres.

11 *Answers:* **1** 0%. **2** 60%. **3** \$25500. **4** \$7522.50. **5** \$25000. **6** 3¢. **7** \$12000. **8** \$12315 \$35685. **9** \$243.20. **10** \$9500.62½ \$5179.74. **11** C. & S. M., 2⅔%. **12** \$19600.

12 *Answers:* **1** \$2900.87. **2** \$506.54. **3** 2666⅔ lb. sugar cane; 4000 lb. beet root; 5000 lb. wheaten flour. **5** 0 100° C. 33⅓° C. 44⅘° C. 68° F. **6** 61.952 gr. **7** 732. **8** $\frac{37}{65}$° C. **9** Germany, 27.6+% England, 12.3+% Ireland, 12.5+% Norway and Sweden, 10.6+%. **10** 2.2+% 42.2+% 17.8+%.

13 The pupils should be led to see by exercises similar to those given on this page that symbols denoting quantity may be expressed by figures and by letters, and that operations may be denoted by signs. The knowledge that pupils possess of the expression of numbers by figures and their operations by signs should be used in leading them to acquire a knowledge of the expression of algebraic quantities and operations. Such knowledge will help them to make generalizations in number and to solve problems that cannot be solved easily by the aid of figures.

14 Pupils will probably be able to perform the first ten exercises with little difficulty. If any difficulty is found, let figures be substituted for letters in representing number, and let the number of exercises be increased.

In clearing equations of fractions, lead the pupils to see the principle involved by using figures instead of letters. The fact that multiplying both sides of the equation by the same number does not affect the equality may be shown by multiplying equal numbers by the same number and letting the pupils see that the products are equal.

In **9**, the axiom that equal quantities divided by the same or equal quantities give equal quotients may also be shown by the use of figures; thus:

$$20 + 10 = 30 \qquad \frac{20+10}{2} = \frac{30}{2}$$

Other axioms may be shown in a similar way as they are needed.

15 *Answers:* **1** Horse, \$288 Cow, \$72. **2** A, \$100 B, \$50 C, \$150. **3** Robert, 40 William, 20 Thomas, 120. **4** 9 oranges and 9 bananas. **5** 12. **6** Father, 48 yr. Son, 24 yr. **7** 8 lb. **8** 4 lb. Mocha 12 lb. Java. **9** A, \$6000 B, \$3000 C, \$1000. **10** A has \$600.

The following form of analysis is suggested for this class of problems:

1 Let $x =$ cost of cow.
$4x =$ " " horse.
$5x =$ " " " and cow.
$5x =$ \$360.
$x =$ \$72, cost of cow.
$4x =$ \$288, cost of horse.

The pupils should learn to select the number that can be most conveniently represented by x. For example, in **2**, the number of dollars that B has, and in **3**, the number of marbles that William has, is the most convenient unit of representation.

In simplifying **11** to **15**, lead the pupils to see how the parentheses may be removed in such examples as: $12 + (4 + 2)$; $(8 + 5) + (6 + 2)$; $(8 - 6) + (8 - 4)$; $16 - (8 - 6)$; $10 - (6 + 2)$. It may be shown that the $10 - (8 - 6)$ is equal to $16 - 8 + 6$ by first subtracting 8 from 16 as indicated, and calling attention to the fact that we have subtracted a number too large by 6, and therefore we must add 6 to the difference. In the expression $10 - (6 + 2)$ by subtracting 6 instead of $6 + 2$, we have a subtrahend too small by 2, and therefore we must subtract 2 from the difference.

16 The four axioms involved in **5** to **8** are as follows:

If the same quantity or equal quantities be added to equal quantities the sums will be equal.

If the same quantity or equal quantities be subtracted from equal quantities the remainders will be equal.

If equal quantities be divided by the same quantity or equal quantity the quotients will be equal.

Two quantities each equal to a third quantity are equal to each other.

The kind of illustrative problems called for is shown in the following:

James has 18 cents, which is 3 cents more than John has. How many cents has John?

Let x = the number of cents that John has.

$x + 3 = 18$, the number of cents that James has.

Subtracting 3 from these equal quantities and there is given:

$$x + 3 - 3 = 18 - 3.$$

$$x = 15.$$

Lead the pupils to give and to solve similar problems illustrating the four principles.

17 *Answers:* **16** 8 6 15. **17** 12 10 4. **18** 12 8 9. **19** 12 18 30. **20** 8 24 15.

In removing the parentheses (**11** to **15**), lead the pupils to see the reason for changing the sign by asking the following questions: "In **11**, what is to be subtracted from 20? If 6 alone is subtracted, is the result larger or smaller than the required answer? What more must be subtracted? How may both processes be indicated?" In the same way proceed with other exercises until the pupils can see why parentheses may be removed by changing the signs of all except the first quantity.

18 *Answers:* **1** Corn, \$1.42; wheat, \$1.54. **2** James, 26 yr.; sister, 16 yr. **3** Robert, 12¢; James, 24¢. **4** 34, 28, 20. **5** 18, 18, 24. **6** 8 lb. @ 7¢, 12 lb. @ 5¢. **7** 7, 8, 9, 10. **8** 16 and 24. **9** 24. **10** 48. **11** 40. **12** Robert, 96¢; Ralph, 72¢. **13** 180 A. **14** 18 yr. **15** 10, 8.

19 *Answers:* **1** Father, 45 yr.; son, 18 yr. **2** \$1200, \$1800, \$2500. **3** *A*, \$4000; *B*, \$6000; *C*, \$9000. **4** \$.250. **5** $13\frac{1}{3}$ lb. **6** 800. **7** $43\frac{7}{11}$. **8** 640. **9** 40. **10** \$5. **11** 8¢. **12** \$800. **13** \$4800. **14** \$8000. **15** \$900. **16** \$70. **17** \$4000.

20 The theory of negative quantities may be shown by the device indicated at the head of the page. In the relative series indicated, all the quantities may be said to increase by 1, or by a from left to right, and to decrease by 1, or by a from right to left, the positive quantities being at the right of zero, and the negative quantities being at the left of zero. The device may be extended in illustrating **1**, the pupils being asked to begin at the zero point and pass the pencil to the right 4 spaces. This would be indicated by $+4a$; then 3 more spaces. The distance from zero would now be indicated by $+7a$; and by moving the pencil over 2 more spaces the distance from zero would be indicated by $+9a$. **2** may be illustrated in the same way, the pencil in each case passing in the opposite or negative direction. The spaces passed over in all would be indicated by $-6a$. In **3**, the pencil starts from zero, as before; passes to the right, as indicated, 4 spaces; then to the left 3 spaces. The pencil now is 1 space to the right of zero, and the distance would be indicated by $+1a$, or $+a$. The pencil moves to the right 2 spaces, and the distance would be indicated by $+3a$, which is the required answer.

Considering subtraction as the process of finding the difference between two quantities, the minuend and subtrahend may be indicated in the relative series at the right or left of zero, and the difference by the distance between them. Thus, in **4**, the distance between $+4a$ and $+3a$ is $+a$. In **5**, the question is, what quantity added to $-2a$ will equal $+4a$. Beginning at $-2a$, the pencil must move to the right, or in a positive direction, 6 spaces before it reaches $+4a$. The difference is, then, indicated by $+6a$. In **6**, the pencil must move 7 spaces to the left from $+3a$ to reach the point $-4a$. The difference is therefore indicated by $-7a$.

After several similar illustrations are given the pupils should be ready to apply the principle expressed in **7**. In Algebra, the multiplier indicates the number of repetitions either of addition or subtraction that is made of the multiplicand. For example, $(+4a) \times (+2)$ means that $+4a$ is to be added 2 times, and $(+4a) \times (-2)$ means that $+4a$ is to be subtracted 2 times. The

answer in the first case is $+8a$, and in the second case $-8a$. Again, adding $-4a$ 2 times, we have for a result $-8a$, and subtracting $-4a$ 2 times, we have $+8a$, *i.e.* $(-4a)\times(+2)=-8a$, and $(-4a)\times(-2)=+8a$.

After this explanation is fully understood by the pupils, and can be given by them in multiplying other quantities, they may make use of the following rule:

The sign of the product of two numbers is plus if the signs of the numbers are alike, and minus if the signs are unlike.

Since Division is the reverse of Multiplication, and the quotient is that number which, multiplied by the divisor, will give the dividend, the way of finding the sign of the quotient will readily appear. The rule for both Multiplication and Division in brief will be:

Like signs give plus; unlike signs give minus.

21 Some teaching may be necessary for a few of these exercises, especially those which involve the multiplication and division of powers of the same quantity. No attempt should be made to simplify results by factoring at this time, but later, if it is thought desirable, the results may be reduced to their simplest forms. In some cases of expressed division, as in **14**, $(6ab)\div(ac)$, the common factor a may be removed, as in similar cases of Arithmetic.

22 *Answers:* **1** $ax+bx$ $ay+by$ $-ax-by$ $-ay-by$. **2** $ax+bx+ay+by$ $ax+by-ay-by$. **3** $ax-bx+ay-by$ $ax-bx-ay+by$. **4** a^2+2a+1 a^2-1 $1-a^2$. **5** $6ax+4bx-8cx$ $-6axy-4bxy+8cxy$. **6** $4a^2x+6a^3-8a$ $4a^2x^2+6a^3x-8ax$ $4a^2x+6a^3-8a-2ax-3a^2+4$. **7** $4a^3b+2a^2b^2+ab^3$ $4a^3b+2a^2b^2+ab^3-4a^2b-2ab^2-b^3$ $-4a^3b-2a^2b^2-ab^3+4a^2b+2ab^2+b^3$. **8** $a^2+2ab+b^2+ac+bc$ a^2+ac-b^2-bc $a^2+2ab+b^2+2ac+c^2+2bc$. **9** $2x^3y-4x^2y^2+xy^3$ $2x^3y-4x^2y^2+xy^3+2x^2y-4xy^2+y^3$ $2x^3y-4x^2y^2+xy^3-2x^2y+4xy^2-y^3$. **10** $x^2+2xy+y^2$ x^2-y^2 $x^3+x^2y+xy^2+y^3$ $x^3+x^2y-xy^2-y^3$. **11** $x^3y+x^2y+x^2y^2+xy^2$ $x^3y^2+x^2y^2+x^2y-xy$ $x^4y^2+x^3y^2$ $x^3y^3+x^2y^3$. **12** $x^4-2x^3-13x^2+14x+24$.

13 $x^2+2xy+y^2$ $x^2-2xy+y^2$ x^2+2x+1. **14** x^2-2x+1 $4x^2+8xy+4y^2$ $4x^2-8xy+4y^2$. **15** $a^3+3a^2+3ab^2+b^3$ $a^3+3a^2b-3ab^2-b^3$ a^3+3a^2+3a+1. **16** $a^2b^2+2abcd+c^2d^2$ $a^2b^2+2ab+1$ $x^4+2x^2y^2+y^4$. **17** $2a^2+2a$. **18** $2x^2+2y^2$. **19** $2a^2b+4ab^2+2b^3$. **20** $b+c+d$ $\frac{b+c+d}{b}$ $\frac{b+c+d}{c}$ $\frac{b+c+d}{d}$. **21** $6a+12b+18$ $a+2b+3$ $a^2+2ab+3a$. **22** 1 $a+b$ $a^2+2ab+b^2$. **23** $a-b$ $a^2-2ab+b^2$. **24** $a+b$ 1. **25** $a-b$ $a+b$. **26** $a^2+2ab+b^2$ $a+b$. **27** $2x+1$ $2x-1$. **28** $1-2x$ $2x-1$. **29** $b-c$ $-b+c$ a. **30** $c+d$. **31** $2+3x-5x^2$. **32** a^2-b^2 a^2+b^2. **33** $4x+3y+4$. **34** $3x-2y+1$. **35** $3x+2y$.

23 *Answers:* **1** 26 $a+b+c$ $6a$. **2** 25 $a+b-c$ $8a$. **3** -10. **4** $a-4b+c+g$. **5** $x-3x^2$. **6** $7x^2+4x+y^2$. **7** $8x+5\frac{1}{2}x^2$. **8** $5x^2+x+2y-15$. **9** x^3+3y^3-2x+2. **10** x^2 x^4 y^6 x^2y^2 x^4y^2 $x^2y^2z^2$ $x^2y^6z^2$ $x^2+2xy+y^2$ $x^2-2xy+y^2$ x^2+2x+1 $x^4+2x^2y^2+y^4$ $9a^2+12ab+4b^2$. **11** Sum of their squares + twice their product Sum of their squares − twice their product. **12** x^6 x^3y^3 $x^3y^3z^3$ $x^6y^3z^3$ $x^3+3x^2y+3xy^2+y^3$ $x^3-3x^2y+3xy^2-y^3$ $8x^3+24x^2+24x+8$ $27x^3-27x^2+9x-1$ $64a^3+96a^2b+48ab^2+8b^3$. **13** Cube of the 1st + 3 times the square of the 1st into the 2d + 3 times the 1st into the square of the 2d + the square of the 2d. Cube of the 1st − 3 times the square of the 1st into the 2d + 3 times the 1st into the square of the 2d − the cube of the 2d. **14** $4x^2+12xy+9y^2$ $9x^2-24x+16$ $x^2+4xy+4y$ $9x^2-36xy+36y^2$. **15** $1+3x+3x^2+x^3$ $8x^3+12x^2y+6xy^2+y^3$ $27x^3-54x^2+36-8$ $x^3-9x^2y+27xy^2-27y^3$. **16** $16x^2+8ax+a^2$ $8x^3-36x^2+54x-27$ $16a^2+24ab+9b^2$ $8a^3-60a^2+150a-125$. **17** $56xy$. **18** $30b^2-4ab-15a^2$. **19** $54a^3b+18b^3$. **20** $-7x^3+15x^2+9x+2+12x^2y-30xy^2+7y^3$. **21** $2\times2\times a\times a$ $2\times2\times2\times a\times a\times a$ $2\times2\times2\times2\times a\times a\times b$ $2\times2\times3\times a\times a\times a\times b\times b\times c$ $(a+1)(a+1)\times m$. **22** $a(x+y)$ $a(ax-y)$ $x\times x\times x(a^2+a+1)(a-1)$ $a\times a(b-x)$. **23** $(a+b)(a-1)$ $(a+b)(a+b)$

$(x+1)(x+1)$ $(x+1)(x-1)$. **24** $(a+b)(a+b)(a+b)(a-b)(a^2+ab+b^2)$ $(a+b)(a^2-ab+b^2)$. **25** $(2a+3b)(2a+3b)$ $(3x+9)(3x+9)$ $(3x+9)(3x-9)$. **26** $2\times2\times ab(2b^2-ab+3a^2)$ $2\times2\times2\times a\times a\times a\times b\times b(2-ab^2+2a)$. **27** $2(x+y)(2x+1)$ $4(x+2y)(2+y)$.

24 *Answers:* **1** $x=8-y$ $x=\frac{10-z}{2}$ $x=12+2y$ $x=2(y+z)$. **2** $x=4y-2$ $x=\frac{48-5y}{6}$ $x=36-2y$. **3** $x=8+b+a$ $x=\frac{3a+2b}{6}$ $x=\frac{45-2a}{18}$. **4** $x=84-a-2b$ $x=\frac{6a-5b}{6}$ $x=4a-3ab$. **5** $x=18-y$ $y=18-x$ $x=2+y$ $y=x-2$ $x=\frac{28-y}{2}$ $y=28-2x$ $x=\frac{12+y}{2}$ $y=2x-12$. **6** $x=\frac{32-2y}{3}$ $y=\frac{32-3x}{2}$ $x=\frac{20+3y}{4}$ $y=\frac{4x-20}{3}$ $x=\frac{40-10y}{3}$ $y=\frac{40-3x}{4}$ $x=\frac{20-y}{2}$ $y=20-2x$. **7** $x=\frac{a+12y}{16}$ $y=\frac{16x-a}{12}$ $x=a-4y$ $y=\frac{a-x}{4}$ $x=\frac{8y-6a+b-1}{4}$ $y=\frac{6a-b+1+4x}{8}$. **8** $x=-4y$ $y=-\frac{6}{4}$ $x=\frac{6y}{7}$ $y=\frac{7x}{6}$ $x=\frac{3ay-16a}{12}$ $y=12x+16a$. **9** $x=4$ $5x=20$. **10** $x=5$ $y=4$. **11** $x=5$. **13** $x=8$ $y=2$. **14** $x=5$ $y=6$. **15** $x=7$ $y=5$. **16** $x=3$ $y=6$. **17** $x=9$ $y=2$. **18** $x=3$ $y=10$.

25 *Answers:* **1** $y=6$ $x=8$. **2** $x=6$ $y=2$. **3** $x=4$ $y=10$. **4** $x=3$ $y=2$. **5** $x=6$ $y=4$. **6** $x=5$ $y=6$. **7** $x=8$ $y=3$. **8** $x=10$ $y=8$. **9** $x=9$ $y=7$ **10** $x=12$ $y=4$. **11** $x=9$ $y=12$. **12** $x=10$ $y=12$. **13** $x=15$ $y=9$. **14** $x=8$ $y=8$. **15** $x=12$ $y=4$.

16 $x=18$ $y=8$. **17** $x=20$ $y=24$. **18** $x=2\frac{8}{11}$ $y=-2$. **19** $x=40$ $y=24$. **20** $x=30$ $y=48$. **21** $x=14\frac{2}{3}$ $y=48$ $z=26\frac{2}{3}$. **22** $x=12$ $y=2$ $z=10$. **23** $x=9$ $y=20$ $z=6$.

26 *Answers:* **1** $x=2$. **2** $y=4\frac{1}{3}$. **3** $y=6$. **4** $y=2$. **5** $x=8$ $y=6$. **6** $x=9$ $y=10$. **7** $x=5$ $y=6$. **8** $x=8$ $y=3$. **9** $x=-3\frac{3}{5}$ $y=4\frac{2}{5}$. **10** $x=9$ $y=3$. **11** $x=8$ $y=12$. **12** $x=7$ $y=2$. **13** $x=12$ $y=10$. **14** $x=15$ $y=18$. **15** $x=34\frac{2}{7}$ $y=22\frac{8}{21}$. **16** $x=1\frac{13}{17}$ $y=-1\frac{7}{34}$. **17** $x=6$ $y=16$. **18** $x=67\frac{1}{2}$ $y=46\frac{1}{2}$. **19** $x=42$ $y=3$. **20** $x=6$ $y=25$. **21** $u=6$ $x=8$ $y=4$ $z=9$. **22** $u=18$ $x=8$ $y=12$ $z=6$.

27 *Answers:* **1** $x=8$ $y=5$. **2** $x=6$ $y=10$. **3** $x=12$ $y=20$. **4** $x=9$ $y=8$. **5** $x=12$. $y=2$. **6** $x=4$ $y=16$. **7** $x=20$ $y=15$. **8** $x=18$ $y=7$. **9** $x=7\frac{7}{15}$ $y=14\frac{2}{15}$. **10** $x=28\frac{4}{5}$ $y=-14\frac{2}{5}$. **11** $x=33$ $y=30$. **12** $x=12$ $y=10$. **13** $x=20$ $y=18$. **14** $x=10\frac{4}{5}$ $y=-13\frac{17}{35}$. **15** $x=0$ $y=15$. **16** $x=20$ $y=10$. **17** $x=21\frac{5}{17}$ $y=13\frac{7}{17}$. **18** $x=30$ $y=24$. **19** $x=8$ $y=12$. **20** $x=12$ $y=2$. **21** $x=9$ $y=4$. **22** $x=22\frac{1}{11}$ $y=16\frac{4}{11}$. **23** $x=21\frac{25}{47}$ $y=20\frac{44}{47}$. **24** $x=12$ $y=20$. **25** $x=6$ $y=8$ $z=16$. **26** $x=5$ $y=30$ $z=10$.

28 *Answers:* **1** $a+b-c$ $a-b-c$ $a-b-c$ $a-b+c$ $a+b-c-d+e$. **2** $a^2+2ab+b^2$ a^2-b^2. **3** $6a^2-5ab+2ac-6b^2+23bc-20c^2$. **4** $5c^2-5c+5cd+13d$. **5** $c+d$ $a-b$. **6** a^2+b^2 $a^4-a^3b+a^2b^2-ab^3+b^4$. **7** $a+b-c+d-3g+h$. **8** $b(a-1)$ $x(x-1)$ $a(x-2y+3z)$. **9** $\frac{2(a^2+b^2)}{a^2-b^2}$ $\frac{4ab}{a^2-b^2}$. **10** $(a+b)^2$ a^2+b^2+2ab $(a-b)^2$ a^2-b^2 $(a+b)\times(a-b)$ $(a+b)^3$. **11** $\frac{m}{a}$ 4 $\frac{a}{12-n}$ $\frac{a}{m+n}$ $\frac{m}{a+1}$. **12** 6 $\frac{b+c}{a}$ $\frac{3}{4}$ -6 $\frac{ab-c}{a}$ $\frac{c-d}{a-b}$. **13** 9 12 2 $\frac{ab}{c}$. **14** $\frac{a}{b+c}$ $\frac{b}{ac}$

a^2b-ac $\frac{a}{ab-bc}$ $116\frac{14}{38}\frac{8}{7}$. **15** $22\frac{1}{2}$ $5\frac{1}{7}$ $\frac{b}{9}$ $\frac{a}{c}$ 12.247+ 12.649+ $5\frac{1}{8}$ $\frac{2b^2}{c}$ $\frac{8c^3}{3b}$.

29 *Answers:* **1** 6 8. **2** 18 8. **3** 20 8. **4** John, 12¢ James, 18¢. **5** Apple, $\frac{2}{3}$¢ orange, $5\frac{2}{3}$¢. **6** $6\frac{2}{3}$, $2\frac{2}{3}$. **7** 15 25. **8** Man, 50 wife, 40 daughter, 10. **9** 20 yr. **10** James, 18 John, 12. **11** Coffee, $24\frac{8}{11}$¢ tea, $65\frac{5}{11}$¢. **12** 11100. **13** A, \$29 B, \$11. **14** 656 apples.

30 *Answers:* **1** \$480. **2** \$540. **3** A, 50 B, 30. **4** 36 da. **5** 2.4 h. **6** 40 persons. **7** A, \$7 B, \$5. **8** Pigeon, 40¢ chicken, 50¢. **9** 10 sheep, 20 calves. **10** \$400. **11** Jane, \$250 Sarah, \$400 Ellen, \$150 Mary, \$200. **12** 15 8.

9 is indeterminate. Other answers are, 5 sheep and 27 calves; 15 sheep and 13 calves; 20 sheep and 6 calves.

31 *Answers:* **1** Apple, 2¢ pear, 1¢ orange, 4¢. **2** \$36000. **3** $13\frac{1}{3}$ da. **4** $6\frac{1}{4}$ $1\frac{1}{4}$. **5** $105\frac{1}{7}$ $78\frac{4}{7}$. **6** \$1500 @ 6%. **7** 8, 12. **9** A, $9\frac{2}{11}\frac{7}{1}$ da. B, $15\frac{2}{25}\frac{1}{1}$ da. C, $8\frac{2}{11}\frac{9}{7}$ da. **10** \$3500 in 4's \$1500 in 5's. **11** $\frac{8}{15}$. **12** 48.

8 is indeterminate, any one of forty answers being correct.

32 *Answers:* **1** Nile, 3000 miles Danube, 1600 miles Amazon, 3600 miles. **2** St. Peter's, 442 ft. Bunker Hill, 215 ft. Washington, 549 ft. **3** 3 h. $16\frac{4}{11}$ min. 6 h. $32\frac{8}{11}$ min. **4** \$1400 \$1600. **5** $14\frac{1}{7}$ h. **6** 8 da. **7** $6\frac{1}{4}$ $1\frac{1}{4}$. **8** $\frac{m(c-b)}{a-c}$. **9** $\frac{al}{m+l}$ $\frac{am}{m+l}$. **10** $\frac{al}{m+l}$ $\frac{am}{m+l}$ $\frac{amo}{n(m+l)}$. **11** $a\left(\frac{100+n}{100}\right)^m$. **12** $x=\frac{am(d-c)}{bc-ad}$ $y=\frac{bm(d-c)}{bc-ad}$. **13** $\frac{ab}{bm+an}$ mo. **14** 12 20. **15** 12 h. $32\frac{8}{11}$ min.

33 *Answers:* **12** 192. **13** 108. **14** 729. **15** 8192. **16** $2\frac{1}{4}$. **17** $1\frac{1}{8}$. **18** .729. **19** .0000078125. **20** $\frac{3}{4}$. **21** 108. **22** 283. **23** $1_{\frac{1}{144}}$. **24** .0225. **25** $.244_{\frac{9}{64}}$. **26** $_{81000}$. **27** $_{216}$. **28** 1600 7056 9801 10000 313600. **29** 698900 106276 474721 998001 1000000. **30** 81796. **31** 20.9764. **32** .00005625. **33** 27.210701. **34** 31640625. **35** 1.10271001. **36** 41371138.5616. **37** $94_{\frac{288}{289}}$. **38** 32768. **39** 88.121125. **40** $5861_{\frac{208}{256}}$. **41** $383328\frac{1}{8}$.

34 The formula for the extraction of the square root of a number can be found by finding the square of the root: $t+u$. Pupils who have taken the algebraic exercises will have no difficulty in this. Others will have to be taught. If the symbols representing any root be found too difficult to work with, let figures representing the root of a given power be used, *e.g.*, the square of $25 = 20^2 + 2 \times (20 \times 5) + 5^2$.

35 *Answers:* **1** 32. **2** 43. **3** 51. **4** 62. **5** 73. **6** 35. **7** 46. **8** 36. **9** 57. **10** 76. **11** 67. **12** 74. **13** 83. **14** 94. **15** 89. **16** 97. **20** 4.2. **21** 6.3. **22** 5.4. **23** 3.5. **24** 5.6. **25** 4.7. **26** 5.8. **27** 6.9. **28** 8.8. **29** 9.6. **30** 9.9. **31** 7.9. **32** 7.9. **33** 231. **34** 342. **35** 254. **36** 364. **37** 286. **38** 523. **39** 475. **40** 279. **41** 634. **42** 488. **43** 596. **44** 678. **45** 969. **46** 804. **47** 709. **48** 232.1. **49** 354.6. **50** 465.8. **51** 50.76. **52** 60.19. **53** 1.41+. **54** 6.32+. **55** 15.55+. **56** 37.14+. **57** 40.92+. **58** 1.87+. **59** 4.02+. **60** 2.69+. **61** 2.84+. **62** 12.66+. **63** 5.72+. **64** 2.19+. **65** 30.009+. **66** 2.23+. **67** 3.61+. **68** 10.83+. **69** 5.22+. **70** .64+. **71** .87+. **72** .73+. **73** .52+.

36 *Answers:* **1** 14 rd. **2** 186 ft. 274 ft. 511.23+ ft. **3** 12.649+ rd. 6.324+ rd. **4** 9.8+ rd. on two sides. **5** 254.13+. **6** 884.84 ft. **7** 148 blocks $98\frac{2}{3}$ ft. square. **8** $95687\frac{1}{2}$ paving stones. **9** 14.14+ in. **10** 24 ft. by 16 ft. **11** 12.29 ft. on longer side 12.11 ft. on shorter side. **12** 154.5 rd. **13** 19 and 15. **14** \$141.40. **15** \$141.40.

37 *Answers:* **7** 80. **8** 32. **9** 41. **10** 45. **11** 68. **12** 63. **13** 99. **14** 97. **15** 4.7. **16** 33. **17** 12.7. **18** .89. **19** 362. **20** 415. **21** 472. **22** 903.

38 *Answers:* **1** 1.58+. **2** 3.91+. **3** 5.64+. **4** 11.69+. **5** 1.68+. **6** 2.43+. **7** 1.89+. **8** 4.82+. **9** 1.91+. **10** 2.38+. **11** .79+. **12** .42+. **13** .76+. **14** .43+. **15** .39+. **16** .74+. **17** 13 in. 35 in. **18** 87 in. 72 in. **19** 12.9+ in. 47.55+ in. **20** 73 in. 28.4+ in. **21** 1.41+ ft. **22** 10.06+rd. by 15.09+ rd. **23** 12.14+ ft. **24** 6 ft. long, 3 ft. wide, $1\frac{1}{2}$ ft. deep. **25** 8.07+ ft. **26** 160 rd. **27** 88.4+in. 132.6+in. 176.8+in. **28** 150.26+rd.

39 The following facts should be drawn from the pupils by teaching, as before shown:

Matter is anything we get a knowledge of through the senses.

A *body* is a limited portion of matter.

Space is the room a body occupies, and the room that is around a body.

A *volume* is a limited portion of space. A volume may be represented by a *solid*, which has three dimensions, length, breadth, and thickness.

A *surface* is the limit of a volume, and has only two dimensions, length and breadth.

A *line* is the limit of a surface, and has only one dimension, length.

A *point* is the limit of a line. It has position only. It can be represented by a dot.

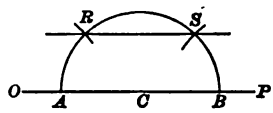
A *straight line* is a line which has the same direction throughout its entire length.

A *curved line* is a line that constantly changes its direction.

(For definitions of *horizontal line* and *vertical line*, see Manual, page 168.)

Lines are *parallel* when they have the same direction. However far prolonged, they can never meet. When one line meets another line so as to make the adjacent angles equal, the lines are said to be *perpendicular* to each other.

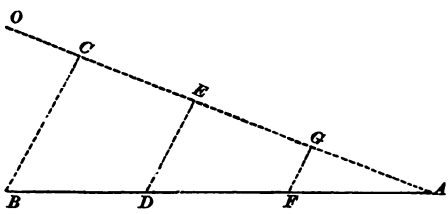
9 A line may be drawn parallel to another line as previously shown, or by the following way:



To draw through the point *R* a line parallel to the given line *OP*.

From the point *C*, with a radius equal to *CR*, draw the semi-circumference *ARB*. From *B* as a center, with a radius equal to *AR*, cut the circumference at *S*. Join *RS*, and we have a line parallel to *OP*. (Let the pupils show why the lines are parallel.)

11–12 For a method of dividing a line into 2, 4, or 8 equal parts, see Manual, page 169. The following method of dividing a line into any number of parts may be taught:



To divide a line *AB* into equal parts.

Draw the line *AO* of any length, and lay off on that line parts of any convenient equal length. Join *BC*, and through the points of division on *AO* draw lines parallel to *BC*. These lines divide *AB* into equal parts.

The standard unit of length in this country is the yard, the same as the imperial yard of Great Britain. Its length is $\frac{3600}{3951}$ of the length of a pendulum which vibrates seconds in a vacuum at the level of the sea at 62° Fahrenheit in the latitude of London. Other interesting facts concerning how and where the standard yard is kept can be gathered from a cyclopedia and given to the pupils.

40 Teach, as before, the following definitions:

An *angle* is the difference of direction of two lines in the same plane. The point where the two lines meet is the *vertex* of the angle. A *right angle* is the difference of direction half as great as oppositeness; or, it is an angle formed by two lines extending perpendicularly from each other. An *obtuse angle* is an angle

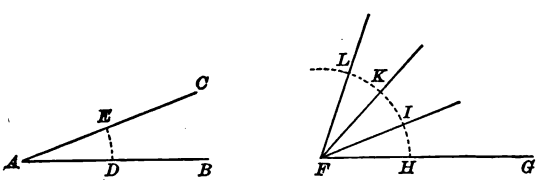
greater than a right angle. An *acute angle* is an angle less than a right angle.

An angle is measured on the circumference of the circle whose center is at the vertex of the angle. The unit of measure is a degree, which is $\frac{1}{360}$ of the circumference of a circle. The following figures will suggest a method of making an angle equal to a given angle, and also for making angles equal to twice or three times the size of a given angle:

13 To draw an angle equal to a given angle.

The pupils will see that the proof of the equality of these angles rests upon the fact that equal arcs subtend equal angles. This is implied in what they have learned about the measurement of angles.

14 To draw an angle twice and three times the size of a given angle.

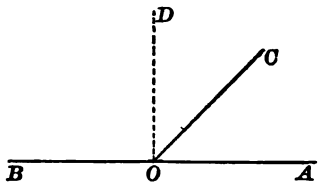


In teaching the above problems, as well as all that follow, the teacher should give as little direct assistance as possible, leading them on slowly by suggestions and questions.

41 *Vertical angles* are angles that have a common vertex, and that have sides extending in opposite directions.

Adjacent angles (*a* and *b* in the figure) are angles that have the vertex and one side common, and whose other sides are opposite parts of the same straight line. (It will be observed that one element of adjacent angles is left out in **2**.)

4 To prove that the sum of two adjacent angles is equal to two right angles.



Draw DO perpendicular to AB.

$AOC + BOC = AOD + BOD$.

$AOD + BOD = 2$ right angles.

$\therefore AOC + BOC = 2$ right angles.

In **7**, lead the pupils to see and to say that the angles a and b (in the figure, **10**) = the angles b and c. Taking away the common angle b, the angle a = the angle c.

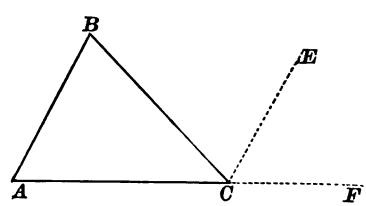
In **10**, the four angles c, d, n, and m are called *internal angles*, because they lie between the parallel lines; and the four angles a, b, o, and p are called *external angles*. The angles a and m and the angles b and n are *exterior-interior* angles. The angles c and m and the angles d and n are *alternate-interior* angles. Lead the pupils to discover what angles are equal, and why they are equal. The lines AB and CD are drawn parallel to each other.

A *plane surface* is such a surface that, if any two points in it be connected by a straight line, that line will lie wholly in the surface.

Let the definitions called for in **12** be concise and comprehensive. (For method of teaching, see Manual, pages 104 and 116.)

42 Let the solution of the theorems and problems be made by measurement and construction, and also by demonstrations so far as the pupils can be led to make them or to understand them. The following figures and hints may suggest demonstrations of the more difficult propositions:

3 Draw CE ∥ to AB.
Prolong AC to F.
$ECF + ECB + ACB = 2$ right angles.
$A = ECF$.
$B = ECB$.



Let the order of demonstrating **5**, **6**, and **7** be reversed.

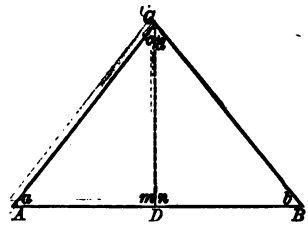
6 Bisect angle C.

$AC = CB$.

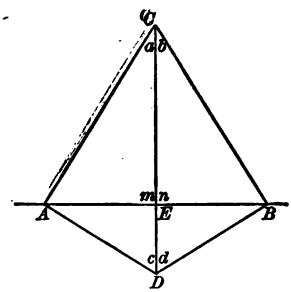
CD common.

$c = d$.

$\therefore a = b$.



From the fact proved in **6**, it can be shown that the angles of an equilateral triangle are equal.



$AC = CB$.

$AD = DB$.

Join AB.

$CAB = ABC$.

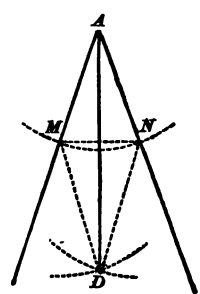
$BAD = ABD$.

$CAD = CBD$.

The exercises from **11** to **15** are to be performed with protractor and ruler.

43 The pupils may be able to solve **3** by demonstration.

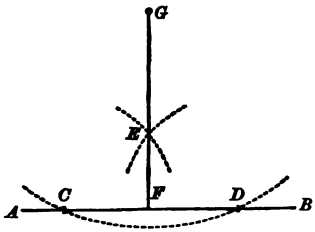
In **4**, lead the pupils to prove that the two triangles are equal, that the angle at the vertex is bisected by the line drawn from the middle point of base to vertex, and that the base is bisected.

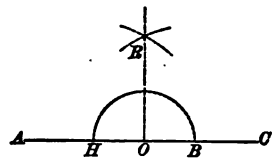


This figure will suggest a method of bisecting the angle A. Lead the pupils to prove that the triangle AMD = the triangle AND, and that therefore the angle A is bisected.

For a method of bisecting a line, see Manual, page 169.

The following figures will suggest ways of solving the problems contained in **7**:



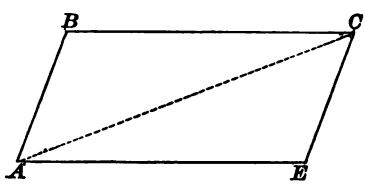


It would be well to apply the principles involved in the last exercises of the page by actually measuring distances in a field. This may be done with lines and stakes or with stakes alone.

44 The definitions called for on this page are supposed to have been taught previously (see Manual, page 104).

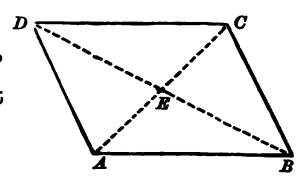
The simplest solution of **3** may be made by dividing a quadrilateral into 2 triangles.

The propositions on this page can all be performed by measurement or construction. As many of them should be demonstrated by the pupils as possible. They will not be found difficult for those who have demonstrated the preceding propositions. The following hints might be given if necessary:



What angles do you know to be equal? What lines? Compare the size of the two triangles. If we were to prolong one of the sides, what two angles are equal to 2 right angles? What other two angles must be equal to 2 right angles?

What angles can you prove to be equal? What lines? What triangles? What other lines?



The pupils may be told that an isosceles trapezoid is a trapezoid whose sides between the two parallel lines are equal. The propositions contained in **13** to **16** can be easily proved.

45 A polygon with two reëntrant angles:

Exercises may be given to show how the area of such a polygon may be found. Let the pupils discover two ways, and what dimensions must be known.

The simplest way to construct a regular polygon is from the circle. Show to the pupils that by means of compasses the circumference of a circle can be divided into any number of equal parts, and that the straight lines connecting the points of division are the sides of a regular polygon.

The formulas called for in **15** and **16** are, $S = ah$; $a = S \div \frac{h}{2}$; $b = S \div \frac{a}{2}$; $a = \sqrt{S}$.

46 The triangles ADE and ABC may be called similar, because they are of the same shape. Later (page 50) the pupils will learn more definitely what similar polygons are. Triangles are equivalent if they have the same size, and equal if they have the same shape and size.

The following formulas should be made by the pupils from their knowledge of finding areas:

Triangle, $S = \frac{ah}{2}$; *trapezoid,* $\frac{a+c}{2} \times h$; *regular polygon,* $S = \frac{pr}{2}$.

The transformation of polygons into equivalent polygons of any required shape can be readily made if the method of finding the areas is thoroughly understood.

47 *Answers:* **1** $\frac{1000}{1089}$ A. **2** 1728 sq. ft. 4290 sq. ft. $25278\frac{3}{4}$ sq. ft. **3** $272\frac{1}{4}$ ft. **4** 80 sq. ft. 2016 sq. ft. **5** 1512 sq. ft. 61640 sq. ft. **6** $217\frac{1}{8}$ ft. **7** 147.5+ ft. **8** 153 sq. ft. **9** 3 ft. 8.3+ in. **10** 344.6+ ft. **11** 141.7+ ft. **12** 4380 sq. ft. **13** $\frac{1}{8}$. **14** 17+ bundles.

48 *Answers:* **5** 5 in. 15 in. **6** 8 in. 13.41+ ft. **7** 25.98+ yd.

The pupils should be encouraged to give other proofs than those here given. The formulas to be given are: $h=\sqrt{b^2+p^2}$; $b=\sqrt{h^2-p^2}$; $p=\sqrt{h^2-b^2}$.

49 *Answers:* **1** 100 ft. **2** 25 ft. **3** 43.8+ rd. **4** 60 m. **5** 62.4+ ft. **6** 120 rd. **7** 19.59+ yd. **11** 34.66+ ft. 693.2+ sq. ft. **12** 25.45+ ft. **13** 70.71+ ft. **14** 24.08+ ft. 32.8+ ft. **15** 45.69+ ft. **16** 30.98+ ft. **17** 36.76+ ft. **18** 65.28+ rd. **19** 25.45+ rd. **20** 26.07+ ft. **21** 722.9+ ft.

50 The term *corresponding* may be used instead of homologous, if preferred. The proof called for in **6** may be experimental rather than demonstrative. It follows what is supposed to be done in previous exercises.

The proof called for in **7** may be made from measurement and comparison of the sides of similar polygons.

51 *Answers:* **1** 9 times as large. **2** 4 : 1. **3** 2 : 1. **4** 5 : 1 25 : 1. **5** 438¾ sq. ft. **6** 94.8+ ft. **7** 5250 sq. rd. **8** 222⅔ sq. ft. **9** 242.4+ ft. **10** 2.4+ ft. **11** 35¾ ft. **12** 36 ft.

These exercises should be performed by proportion. It would be well for pupils to write the proportions and statements in full; thus, in **5**:

(40 ft.)² : (50 ft.)² = 300 sq. ft. : number of sq. ft. in larger triangle.

$$\frac{300\times 2500}{1600}=\text{number of sq. ft. in larger triangle.}$$

and in **6**:

4000 sq. ft. : 10000 sq. ft. = 60² : the square of the side of larger hexagon.

$$\frac{10000\times 3600}{4000}=\text{the square of the side of larger hexagon.}$$

$$\sqrt{\frac{10000\times 3600}{4000}}=\text{side of larger hexagon.}$$

52 *Answers:* **1** 33 ft. **2** 93⅓ ft. **3** 27⅞. **4** 29 $\frac{5}{11}$ ft. **6** 86⅔ ft. **7** 43⅛ ft.

In **8**, a line may be drawn downward from B parallel to AX, and one from A parallel to BX, so as to meet the first line at E. Thus would be found two equal triangles, two angles and included side of one triangle being equal to two angles and included side of the other.

In **9**, extend the line XA to a point F. Draw FG perpendicular to FX. Join GX. From the point A draw a line AH parallel to GX. Thus are formed two similar right-angled triangles, and $FH : FG = FA : FX$.

53 In **1**, the lines AX and AY are found by the method shown in **5**, page 52. Ax is the same fractional part of AX that Ay is of AY. The two triangles Axy and AXY are similar, and $Ax : AX = xy : XY$.

The following definitions may be taught as previously shown:

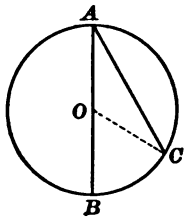
A *circle* is a plane figure bounded by a curved line, all points of which are equally distant from a point within, called the center. The *circumference* of a circle is the line which bounds it. The *diameter* is a straight line passing through the center and terminating at the circumference. The *radius* is a straight line connecting the center with any point in the circumference. An *arc* is any portion of a circumference. A *chord* is a straight line connecting the extremities of an arc. A *segment* is a portion of a circle bounded by a chord and its arc. A *sector* is a portion of a circle bounded by two radii and the included arc. A *semi-circle* is a portion of a circle bounded by a diameter and half the circumference.

Draw lines as indicated in **10**, and show that every point of the perpendicular is equidistant from the ends of the chord. **11** and **12** can be readily shown from **10**.

54 In **1**, join the points by lines, which may be regarded as chords of the required circle, and erect perpendiculars from the middle points.

In drawing an inscribed angle (**4**), there are three possible conditions. The center may be in one of the sides of the angle, or, between the sides of the angle, or, without the sides of the angle. The first case should be proved first, the proof resting upon the

facts, (1) that the angle BOC is equal to the sum of the angles A and C; and (2) that the angles A and C are equal. From this proof the other cases may be easily proved.



A *secant* is a straight line cutting the circumference of a circle in two points.

A *tangent* is a line which touches a circumference in one point without cutting it.

The proof of **6** rests upon the facts, (1) that a line from the point of contact to the center is the shortest distance between the tangent and the center; and (2) that the shortest line between a point and a straight line is the perpendicular line.

In **8**, connect the point with the center of the circle, and, upon this line as a diameter, describe a circle which will cut the circumference of the first circle at two points. Join these two points with the given point outside of the circumference, and the lines thus drawn are the tangents required. This can be easily proved from what has been proved before.

Before giving **9** to **12**, show that a circle is inscribed in a polygon when its circumference touches every side of the polygon, and that a circle is circumscribed about a polygon when its circumference passes through all the corners of the polygon.

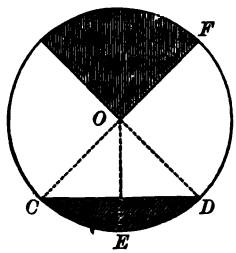
In **9**, the point of intersection of the lines bisecting the angles is the center of the inscribed circle. To prove that the sides of the triangle are tangents to the circle, draw lines from the center perpendicular to the sides of the triangle, and show from the equivalence of the triangles that the perpendicular lines are equal, and are therefore the radii of the circle touching all sides of the triangle.

In **10**, draw such perpendiculars to radii as will intersect and form a triangle.

11 and **12** can be readily done when it is understood that the bisectors of the angles of a regular polygon meet in a point equally distant from all the sides and all the corners. This can be proved from the equality of the triangles.

In **15**, the pupils can readily find the approximate ratio by measurement. They can also estimate by calculation the perimeter of a regular polygon inscribed in a circle having a given radius, and can see that the greater the number of sides of the inscribed polygon, the nearer the perimeter approaches the length of the circumference.

55 From what has been done previously (see Book VI., page 18) the formulas in **1** to **5** can be illustrated.



6 The sector *COD* may be divided into any number of triangles whose altitude is *EO* and the sum of whose bases is *CD*.

7 The area of the segment *CED* is equal to the sector *COD* — the triangle *COD*, *i.e.*

$$CED \times \frac{EO}{2} - CD \times \frac{FO}{2}.$$

To find the area of a circular zone or circular ring, subtract from the area of the circle the part not included in the zone or ring. Let the pupils determine what dimensions must be given.

Let the proofs called for in **10** to **13** be made from construction and comparison of measurements. Pupils may be led to use what they already know of similar polygons.

To draw an ellipse, stick two pins or tacks into the surface upon which the ellipse is to be drawn, and tie to them a string longer than the distance between them. Describe with a pencil the curve, keeping the string constantly stretched to its full length. The points *A* and *D* in the figure are the *foci*, *CD* the *major axis*, *AB* the *minor axis*. The *eccentricity* is the ratio that the minor axis bears to the major axis. An ellipse is a plane figure bounded by such a curved line that if from any point in it straight lines be drawn to two points within, called the foci, their sum will be a constant quantity.

56 *Answers:* **9** 175 sq. in. 124 cu. in. **10** 600 sq. in., 1000 cu. in. $253\frac{1}{2}$ sq. ft., $274\frac{5}{8}$ cu. ft. $80\frac{2}{3}$ sq. ft., $49\frac{8}{27}$ cu. ft. **11** $4\frac{1}{27}$ in. **12** 4.4+ ft. **18** 384 sq. in. **19** 23.34+ cu. ft. **20** $493\frac{1}{8}$ cu. in.

This and the following page of exercises should be taught largely from the blocks. The following notes apply to some of the more difficult points:

A *dihedral angle* is the opening between two intersecting planes. A *trihedral angle* is the opening of three planes which meet at a common point.

A *tetrahedron* is a solid bounded by four planes. A *regular polyhedron* is a polyhedron having equal and regular faces and equal polyhedral angles. A *prism* is a polyhedron bounded by parallelograms and two equal and parallel polygons, called bases. A *right prism* is a prism whose lateral edges are perpendicular to the bases. A *parallelopiped* is a prism all of whose faces are parallelograms. A *pyramid* is a polyhedron bounded by triangles that have a common vertex, and by a polygon, called the base. A *regular pyramid* is a pyramid whose base is a regular polygon, and whose vertex is directly above the center of the base. A *quadrangular pyramid* is a pyramid whose base is a quadrilateral. For other suggestions relating to pyramids, etc., see Manual, page 171, and for rules called for, see Appendix, page 115.

57 *Answers:* **11** 207.3456 sq. ft. 226.1952 cu. ft. 502.656 sq. in. 4021.248 cu. in. **12** 10.472 sq. ft. 28.4925+ sq. ft. **13** 3.5342 cu. ft. 41.4517 cu. ft. **14** 2869.0858 cu. in. **15** 804.2496 sq. in. 2144.6656 cu. in. 226.9818 sq. ft., 321.5575 cu. ft. 2576.896 sq. ft., 10761.79392 cu. ft. **16** 25.4+ in. **17** 6 sq. ft. 2.76+ ft. 8.676+ ft. **18** 10.4+ ft. **19** 2.01+ in. **20** 32.127+ gal.

A *sphere* is a volume that may be generated by the rotation of a semi-circle upon the diameter as an axis. The *diameter* of a sphere is a straight line passing through the center and having its extremities in the surface. A *great circle* of a sphere is a section made by cutting the sphere through the center. A *small circle* of a sphere is a section made by cutting the sphere outside the center. A *spherical zone* is the surface of a sphere included between the circumferences of two parallel circles, or between the circumference of a circle and a parallel plane tangent to the sphere.

A *spherical segment* is a portion of a sphere included between two parallel circles, or between a circle and a parallel plane tangent to the sphere. A *spherical sector* is the portion of a sphere which may be generated by the rotation of a sector of a circle upon a diameter of the sphere as an axis.

Notes applying to other exercises on this page will be found on pages 172 and 173 of the Manual.

58 *Answers:* **5** 499.2 lb. 1684.8 lb. 1.11+ ft. 2.52+ ft. **6** 8 : 1. **7** 5 ft. **8** 2 ft. **9** 27 cubes. **10** 12.7 ft. **11** $118\frac{37}{80}$ bu. 6 ft. 3.4+ in. **12** 13.3+ ft. **13** 4.3+ ft. × 6.4+ ft.

59 *Answers:* **1** 10.122+ ft. 15.69+ ft. 12.4+ ft. 10.5+ ft. **2** (*a*) 80 ft.; (*b*) 1432+cu. ft.; (*c*) 14.9+ cords. **3** 2010624000000 sq. mi. 25673657856000000 sq. mi. **4** 9 times 27 times. **5** Moon = .001801824 of earth Earth = $\frac{1}{144\,28\,97}$ of the sun. **6** 9.4+ in. **7** Surface, 432 sq. in. Volume, 610.56+ cu. in. **8** 3.04+ in. 2.+ in., 2.5+ in., 10.39+ in. **9** Diameter, 11.6+ in. Depth, 5.6+ in. **10** 624 balls. **11** 3040+ cu. in. **12** 6.075 T. **13** 17.7+$^{\text{cu m}}$.

60 *Answers:* **1** (*a*) 12 ft.; (*b*) 12 ft.; (*c*) $8\frac{1}{3}$ ft.; (*d*) $11\frac{7}{16}$ ft. **2** (*a*) $13\frac{1}{8}$ ft.; (*b*) $12\frac{19}{48}$; (*c*) $29\frac{17}{32}$ ft. **3** \$41.85. **4** 4819.5 ft. **5** \$27.18. **6** $10\frac{1}{8}$ bundles \$2.83½ (allowing ½ of openings). **7** 58 bundles. **8** (*a*) $2401\frac{3}{4}$ ft.; (*b*) $2202\frac{1}{3}$ ft.; (*c*) 1968 ft.; (*d*) \$74.78; (*e*) \$193.36.

61 *Answers:* **1** 450 sq. ft. 2700 sq. ft. **2** $2\frac{15}{23}\frac{23}{04}$ C. **3** $14\frac{49}{180}$ C. **4** $58\frac{83}{99}$. **5** \$42. **6** 113 cd. **7** 7104 bricks \$99.46. **8** 195840 bricks 172836 bricks. **9** 36.35 gal.

62 *Answers:* **1** (*a*) 109494 bricks; (*b*) \$1067.57; (*c*) 130 bundles; (*d*) 8816 sq. ft. **2** (*a*) \$8.17; (*b*) \$6.48; (*c*) \$15.44; (*d*) 18 rolls; (*e*) $39\frac{1}{18}$ yd. **3** \$18.50. **4** $31\frac{1}{8}$ yd. $29\frac{17}{18}$ yd. **5** 15 rolls \$23.94 \$5.30.

63 *Answers:* **1** About 15 T. **2** 2.15+T. **3** 5.4+T. **4** 10.9+ ft. **5** $9\frac{13}{27}$ ft. **6** 422.4 cd. 1267.2 bu. 422.4 cu. yd. **7** 7480 gal. **8** 97.9+ bbl. **9** 154.9+ lb. **10** About 42 C. \$297.60. **11** 697.5 T. $3555\frac{5}{9}$ sq. yd. **12** \$24.92.

64 *Answers:* **2** $38\frac{8}{9}$ bd. ft. **3** $82\frac{1}{2}$ ft. **4** 935 ft. **5** $37\frac{1}{7}\frac{1}{3}$ ft.

65 *Answers:* **1** 57 sq. ft. \$4.50 $47\frac{5}{8}$ sq. ft. 2938 packages. **2** 848 boxes. **3** 41.5 sq. ft. \$11.14.

66 *Answers:* **2** 63+ ft. $10\frac{1}{2}$ 8 49.6+ sq. ft. 8¢.

67 *Answers:* **1** 14.18+ sq. ft. 60.13+ sq. ft. **2** 50.26+ sq. in. 201.06+ sq. rd. **3** 14.28+ rd. 25.72+ rd. **4** 19.635 sq. ft. 10.90+ sq. ft. 4.36+ sq. ft. 17.45+ sq. ft. **5** 80 rd. **6** \$70.20. **7** $5\frac{9}{3}\frac{9}{3}$ ft. **8** 60 yd. 58.7+yd. **9** Each side, 35.3+ft. **10** Length of parts, 96 ft. and 54 ft. Area, 1944 sq. ft. and 3456 sq. ft. **11** 1413.72 sq. ft. **12** \$1140.02. **13** 31.8 lb. **14** 51.9 lb.

68 *Answers:* **1** 60 ft. **2** 2400 sq. yd. **3** 1692 sq. ft. **4** Each side, 50 yd. Altitude, 48 yd. **5** Surfaces, 1 to 4; volumes, 1 to 8. **6** $\frac{1}{3}\frac{9}{7}$. **7** 2 in. **8** 8 times as much. **9** \$5.18. **10** 25.1328 in. **11** 58.8+ ft. **12** 157.08 sq. ft. **13** 17.7+ ft. **14** 42.423+ ft. **15** Triangle, 996 sq. ft.; square, 1296 sq. ft.; circle, 1574.48 sq. ft. **16** \$90.80. **17** Volume of frustum, $\frac{3}{8}\frac{7}{4}$ of cone.

69 *Answers:* **1** 5026.56 sq. ft. 11309.76 sq. ft. 485.43+ sq. in. 67.5+ sq. ft. **2** 7.0+ ft. **3** 2901.6+ sq. ft. **4** 1890.7+ ft. **5** 4.9+ tons. **6** 12.4+ ft. × 38.3+ in. × 22.1+ in. **7** 794596 sq. ft. **8** 12 shingles 8 shingles 64+ bundles. **9** 8.485 rd. **10** 1022.9+^{m}. **11** 64 rd. 6 ft. **12** 10 mi. 3 hr. 32 min.

70 *Answers:* **1** 65.45 cu. in. **2** 20.6+. **3** 1.49+ sq. in. **4** 196.7 ft. **5** 170.3 ft. **6** \$44.44. **7** 51.6 ft. 24510 ft. **8** $3\frac{3}{8}$ ft. **9** 15.5+ in. **10** 460.1 ft. 4 A. $135\frac{17}{160}$ sq. rd. **11** 2261.95+ sq. ft. 537.21+ sq. ft.

71–73 Nearly all these exercises are a review of business exercises given in Books VI. and VII. Lead the pupils to give concrete examples before definitions. In some cases, as in describing the various features of a promissory note, it would be well to have the examples written out in full.

In some States no days of grace are allowed, and in some States days of grace are allowed only under certain circumstances. The

teacher should ascertain the law and practice in this regard, and explain to the pupils.

A *collateral note* is one given with stocks, bonds, or other property as security, empowering the payee to sell the same if the note is not paid when it becomes due. An *accommodation note* is one for which the maker receives no consideration. It is given for the purpose of lending credit to the payee. Forms of these notes can be obtained at a bank.

The endorser of a note makes himself responsible for its payment, unless he writes the words "without recourse" before his name.

In answer to **7**, page 73, there may be given examples of general partnership, in which the partners have the same or different capital, and examples of limited partnership, in which the responsibility of one or more of the partners is limited to the amount invested. The general custom of merchants as well as laws of the State regulating partnerships should be ascertained and explained to the pupils.

74–75 The rulings of the cash account should be made as indicated. After the form given on page 75 is carefully looked over by the pupils, they may be asked to write out the account in full. The balance Sept. 21 is $246.61.

76–78 These items should be used in writing cash and personal accounts, as previously shown. Lead the pupils by abundant examples to learn the use of the terms debit and credit, debtor and creditor. A person who receives may be called a debtor, and one who gives, a creditor. It may be helpful for pupils in determining which side of the cash account certain transactions shall be placed to apply the same distinction of receiving and giving to the money-drawer or cash-box. What is in the box in opening the account and what is put into it are to be debited to the account; what is taken out of the box and what remains in balancing the account are to be credited.

The items given on page 76 should be written out by the pupils in the following form:

Dr.			CASH.					Cr.	
Sept. 1	To balance,		192	00	Sept. 2	By 100 bbl. apples,	1.60	160	00
" 4	" 3 gal. P. R. molasses,	.58	1	74	" "	" 200 lb. Vt. butter,	.24	48	00
" "	" 12 lb. Mocha coffee,	.32	3	84	" 5	" int. on $300, 5 mo. 15 da.		8	25
" 6	" 25 bbl. apples,	2.24	56	00	" 7	" water bill,		12	00
" "	" 50 lb. Vt. butter,	.30	15	00	" 12	" 2 cd. oak wood,	5.33	10	66
" "	" 3 bottles chow-chow,	.85	2	55	" 14	" balance,		238	93
" 8	" 40 bbl. apples,	2.00	80	00					
" "	" 60 lb. Vt. butter,	.32	19	20					
" "	" 5 cans Cal. peaches,	.35	1	75					
" 9	" int. on $240, 6 mo. 10 da.		7	60					
" 11	" sales to date,		43	76					
" 13	" 20 bbl. apples,	2.08	41	60					
" "	" 8 " "	1.60	12	80					
			477	84				477	84
Sept. 15	To balance,		238	93	Sept. 18	By gas bill,		7	20
" 16	" 70 lb. Vt. butter,	.34	23	80	" 20	" gift to poor family,		10	00
" 19	" 2 bbl. St. Louis flour,	5.25	10	50	" 22	" balance,		260	29
" "	" 3 qt. cranberries,	.13		39					
" "	" 6 cans tomatoes,	.16		96					
" "	" 3 lb. currants,	.18		54					
" 21	" 2½ lb. sage cheese,	.18		45					
" "	" 8 doz. R. I. eggs,	.24	1	92					
			277	49				277	49
Sept. 23	To balance,		260	29					

The accounts on this and the following page are based upon the items given on page 77.

Dr. CASH. *Cr.*

Sept. 1	To balance,		230	00	Sept. 7	By 3 boxes steel pens,	.62	1	86
" 6	" 3 bbl. Minn. flour,	5.75	17	25	" "	" 2 reams note paper,	2.50	5	00
" "	" 25 lb. B. H. sugar,	.05	1	25	" "	" 5 pkg. stamped envelopes,	.75	3	75
" "	" 10 gal. vinegar,	.20	2	00	" 11	" blacksmith's bill,		23	50
" 7	" sales to date,		22	15	" 16	" 1 wheelbarrow,		2	50
" 12	" 15 lb. Rangoon rice,	.09	1	35	" 19	" balance,		289	74
" "	" 26 gal. kerosene,	.11	2	86					
" "	" 8 lb. Dutch cheese,	.22	1	76					
" "	" 23 pkg. flower seeds,	.08	1	84					
" 15	" 6 lb. Vt. butter,	.28	1	68					
" "	" 7 lb. sage cheese,	.18	1	26					
" "	" 3 gal. syrup,	.75	2	25					
" 18	" 3 lb. crackers,	.14		42					
" "	" 12 lb. Mocha coffee,	.30	3	60					
" "	" 3 boxes P. & C. sardines,	.63	1	89					
" "	" 5 " Keen's mustard,	.25	1	25					
" "	" 4 doz. lemons,	.28	1	12					
" 19	" sales to date,		32	42					
			326	35				326	35
Sept. 19	To balance,		289	74					

Dr. AUSTIN FLINT. **Cr.**

Sept. 2	To balance,		53	70	Sept. 4	By 5 days' work,	1.75	8	75
" 5	" 2 lb. Formosa tea,	.63	1	26	" 8	" 3 loads gravel,	.75	2	25
" "	" 3 lb. Rio coffee,	.32		96	" 20	" balance,		60	77
" "	" 4 lb. Malaga raisins,	.18		72					
" 8	" 1 bbl. St. Louis flour,		6	13					
" "	" 2 gal. N. O. molasses,	.58	1	16					
" "	" 3 bags Indian meal,	2.17	6	51					
" 13	" 17 lb. buckwheat,	.05		85					
" "	" 2 doz. eggs,	.24		48					
			71	77				71	77
Sept. 20	To balance,		60	77					

Dr. HIRAM BOND. **Cr.**

Sept. 9	To 2 bu. N. S. potatoes,	.92	1	84	Sept. 2	By balance,		12	75
" 21	" balance,		45	31	" 6	" 3 pair brogans,	1.75	5	25
					" "	" 2 " ladies' slippers,	1.25	2	50
					" 9	" 3 " children's shoes,	2.13	6	39
					" "	" 5 " rubbers,	.65	3	25
					" "	" 2 " calf boots,	3.25	6	50
					" 14	" 2 " ladies' gaiters,	3.63	7	26
					" "	" 1 " rubber boots,		3	25
			47	15				47	15
					Sept. 21	By balance,		45	31

78 The items on the debit side of the cash account are \$137.50, \$2.40, \$3.70, \$2, \$12.75, \$62.50, \$4.96, \$3.60, \$1.26, \$27.90, \$3.72, \$6.60, \$.54, \$1.15, and \$3. The items on the credit side are \$13.50, \$3.36, \$6.60, \$7.38, \$5, \$27.62, and a balance of \$210.12, making a total footing of \$273.58. The balance in A's account is \$62.28 (debit side), and in B's account, \$13.50 (credit side). The last questions may be postponed until after other accounts have been written.

79–83 In this exercise, and in each of the seven following exercises, it will be found convenient to have the pupils use a sheet of letter size paper having twenty-nine lines, the first and second pages to be used as Day Book, and the third and fourth pages as Cash Book and Ledger. It may be well to go over the items of this exercise and explain each one not fully understood. When this is done, let the pupils rule their papers and make out the account in full before comparing it with the account given.

The following are the missing items of pages 82 and 83, given in the order omitted:

Cash Dr., Jan. 15, To A. Lawrence on $^{8}/_{0}$, \$20.00; Jan. 25, To A. Lawrence, \$6.96. Cash Cr., Jan. 31, By balance, \$51.02. Footing of Cash account, \$109.77. Amos Lawrence Dr., Jan. 12, To mdse., \$2.90. Amos Lawrence Cr., Jan. 15, By cash on account, \$20.00; Jan. 25, By cash in full, \$6.96. Footing of Amos Lawrence's account, \$46.96.

Charles Smith Dr., Jan. 13, To mdse., \$1.50; Jan. 18, To mdse., \$4.55. Charles Smith Cr., Jan. 17, By labor, \$5.25; Jan. 18, By wood, \$13.50; Jan. 31, By balance, \$23.09.

Balance Sheet Dr., Jan. 31, To cash on hand, \$51.02; Jan. 31, To C. Smith, \$23.09. Balance Sheet Cr., Jan. 31, By H. Brown, \$18.00; By net capital, \$1456.11.

Net capital Jan. 31, \$1456.11. Net capital Jan. 1, \$1319.71.

84 On the following four pages are the Day Book and Ledger accounts from these items.

Day Book.

1

West Newton, May 1, 1889.

		Mdse. on hand,		*500*	*00*		
C.B.		*Cash* "		*42*	*00*	*542*	*00*
		——— *4* ———					
	4	*A. A. Evans,*	*Dr.*				
		To 15 gal. N. O. molasses,	*.60*	*9*	*00*		
		" *4 lb. Formosa tea,*	*.75*	*3*	*00*	*12*	*00*
		——— *7* ———					
	4	*Hiram Carter,*	*Dr.*				
		To 2 lb. Vt. butter,	*.25*		*50*		
		" *4 lb. Persian dates,*	*.15*		*60*	*1*	*10*
		——— *Cr.* ———					
C.B.	*4*	*By cash on* $^{2}/_{6}$				*5*	*00*
		——— *8* ———					
		A. A. Evans,	*Cr.*				
C.B.	*4*	*By cash on* $^{2}/_{6}$				*6*	*00*
		——— *Dr.* ———					
	4	*To 2 bu. potatoes,*	*.80*			*1*	*60*
		——— *11* ———					
	4	*Hiram Carter,*	*Dr.*				
		To 3 lb. Rio coffee,	*.25*		*75*		
		" *1 box sardines,*			*50*		
		" *3 lb. Smyrna figs,*	*.20*		*60*		
		" *10 lb. B. H. sugar,*	*.08*		*80*	*2*	*65*
		——— *Cr.* ———					
	4	*By 2 days' labor,*	*1.87*	*3*	*74*		
		" *2 loads gravel,*	*.63*	*1*	*26*	*5*	*00*

Day Book.

2

West Newton, May 15, 1889.

	4	*A. A. Evans,*	*Dr.*				
		To 2½ lb. Mocha coffee,	*.32*		*80*		
		" 8 lb. Rangoon rice,	*.10*		*80*		
		" 4 lb. Malaga raisins,	*.18*		*72*	*2*	*32*
		17					
	4	*Hiram Carter,*	*Dr.*				
		To 3 lb. currants,	*.15*		*45*		
		" 1 bbl. St. Louis flour,		*6*	*75*	*7*	*20*
		Cr.					
C.B.	4	*By cash on a/c*		*15*	*00*		
		" 1½ cd. chestnut wood,	*5.00*	*7*	*50*	*22*	*50*
		18					
	4	*A. A. Evans,*	*Cr.*				
C.B.		*By cash on a/c*				*20*	*00*
		20					
	4	*Hiram Carter,*	*Cr.*				
		By 2 days' labor,	*1.38*			*2*	*76*
		22					
	3	*David Grant,*	*Dr.*				
		To 2 doz. bananas,	*.50*	*1*	*00*		
		" 3 gal. P. R. molasses,	*.65*	*1*	*95*	*2*	*95*
		Cr.					
	3	*By 25 gal. vinegar,*	*.19*			*4*	*75*
		28					
		Hiram Carter,	*Dr.*				
C.B.	4	*To cash on a/c*				*12*	*50*

Ledger.

3

Dr. CASH. *Cr.*

May	*1*	*To balance,*		*42*	*00*	*May*	*12*	*By 10 yd. calico,*	*.10*	*1*	*00*
"	*6*	" *1 doz. oranges,*			*25*	"	"	" *15* " *drilling,*	*.12*	*1*	*80*
"	"	" *3* " *lemons,*	*.20*		*60*	"	"	" *20* " *sheeting,*	*.08*	*1*	*60*
"	*7*	" *Hiram Carter,*		*5*	*00*	"	*20*	" *4 pair brogans,*	*2.25*	*9*	*00*
"	*8*	" *A. A. Evans,*		*6*	*00*	"	*28*	" *Hiram Carter,*		*12*	*50*
"	*17*	" *Hiram Carter,*		*15*	*00*	"	*30*	" *balance,*		*71*	*41*
"	*18*	" *A. A. Evans,*		*20*	*00*						
"	*25*	" *2 lb. cheese,*	*.20*		*40*						
"	"	" *3 lb. nails,*	*.12*		*36*						
"	"	" *4 gal. oil,*	*.15*		*60*						
"	"	" *3 boxes mustard,*	*.22*		*66*						
"	"	" *2 large tubs,*	*.92*	*1*	*84*						
"	*29*	" *2 bottles olives,*	*.83*	*1*	*66*						
"	"	" *6 lbs. lard,*	*.13*		*78*						
"	"	" *3 doz. eggs,*	*.24*		*72*						
"	"	" *2 gal. syrup,*	*.72*	*1*	*44*						
				97	*31*					*97*	*31*
June	*1*	*To balance,*		*71*	*41*						

Dr. DAVID GRANT. *Cr.*

May	*22*	*To mdse.,*	*2*	*2*	*95*	*May*	*22*	*By mdse.,*	*2*	*4*	*75*
"	*30*	" *balance,*		*1*	*80*						
				4	*75*					*4*	*75*
						June	*1*	*By balance,*		*1*	*80*

Ledger.

4

Dr. A. A. Evans. Cr.

May	4	To mdse.,	1	12	00	May	8	By cash on a/c	1	6	00
"	8	" "	1	1	60	"	18	" " "	2	20	00
"	15	" "	2	2	32						
"	30	" balance,		10	08						
				26	00					26	00
						June	1	By balance,		10	08

Dr. Hiram Carter. Cr.

May	7	To mdse.,	1	1	10	May	7	By cash on a/c	1	5	00
"	11	" "	1	2	65	"	11	" sundries,	1	5	00
"	17	" "	2	7	20	"	17	" "	2	22	50
"	28	" cash on a/c	2	12	50	"	20	" labor,	2	2	76
"	30	" balance,		11	81						
				35	26					35	26
						June	1	By balance,		11	81

Dr. Balance Sheet. Cr.

May	31	To mdse. on hand,	Inv't	585	00	May	31	By David Grant,	3	1	80
"	"	" cash "	C.B.	71	41	"	"	" A. A. Evans,	4	10	08
						"	"	" Hiram Carter,	4	11	81
						"	"	" net capital,		632	72
				656	41					656	41
May	31	Net capital,		632	72						
"	1	" "		542	00						
"	31	" gain,		90	72						

85 These transactions are supposed to extend over one month, the account in the Day Book opening Apr. 1 with the first four items, as shown on page 80. The Day Book entries, as before, should occupy two pages of letter size paper, the first page ending with debiting Charles Smith (C) to 5 boxes of sardines. There should be eight entries on each of the two pages of the Day Book. Let the dates of the Day Book entries be scattered through the month at convenient times, omitting Sundays. The items "sales to date" should be written only in the Cash account, being supposed to be one week apart. The Cash account, occupying 17 lines, and Charles Smith's account, occupying 7 lines, should be written on the third page of the sheet. This arrangement permits space for beginning a new cash account May 1 ($714.85), and also allows 3 or 4 lines not to be written upon. On the fourth page of the sheet the account of Amos Lawrence (A) should be written, and also the following closing statements:

Dr. BALANCE SHEET. *Cr.*

Apr. 30	*To mdse. on hand,*	*2000*	*00*	*Apr. 30*	*By Amos Lawrence,*	*53*	*82*
" "	" *cash* "	*714*	*85*	" "	" *net capital,*	*2661*	*03*
		2714	*85*			*2714*	*85*

Net capital, April 30, is $2661.03
" " " *1, was* 1526.16
Net gain, $1134.87

86 This account is supposed to extend from May 2 to May 20. The last items of the first page of the Day Book should appear as follows:

		9					
		George G. Gates,	*Cr.*				
	4	*By 3 European larch,*	*1.12*	*3*	*36*		
	4	" *2 Wisconsin willow,*	*.62*	*1*	*24*	*4*	*60*
		Dr.					
C.B.	*4*	*To cash on* $\%_0$				*10*	*00*

On the second page of the Day Book there will be the accounts of Gates, 5 lines; Hart, 2 lines; Gates, 4 lines; Eaton, 2 lines; Hood, 4 lines; and Gates, 4 lines. The cash account on the third page of the sheet will occupy 22 lines, and give room on the page for the Ledger accounts of Isaac Hart and A. M. Eaton. Cash is credited with the following amounts: \$4.50, \$9.66, \$3, \$2.50, \$3.04, \$10.48, \$2.52, \$10, \$4, \$1.68, \$1.52, \$30, \$2.16, \$3.90, \$4.74, \$1.76, \$1.24, \$.58, and balance on hand May 20, \$298.43. On the fourth page of the sheet will appear the Ledger accounts of Horace Hood, 6 lines, and G. G. Gates, 9 lines, and the Balance Sheet. The debit side of the Balance Sheet will be: Mdse. on hand, \$1000; Cash on hand, \$298.43; Horace Hood, \$98.30. The credit side will be: A. M. Eaton, \$59.25; G. G. Gates, \$27.74; Net capital May 20, \$1309.82, making a total footing of \$1396.81. The net gain (\$139.57) should be noted as before.

87 The entries of first five dates should be written on the first page of Day Book. The others, beginning with the transactions with Charles Smith (C), will just fill the second page. The Cash account (20 lines) and Amos Lawrence's Ledger account (5 lines) will fill the third page of the sheet. On the fourth page will be written the other two Ledger accounts and the Balance Sheet, which will appear as follows:

Dr. BALANCE SHEET. *Cr.*

May 31	*Cash on hand,*	*C.B.*	*74*	*70*	*May 31*	*Chas. Smith,*	*4*	*13*	*40*
" "	*Mdse.* "	*Inv't*	*700*	*00*	" "	*Net capital,*		*961*	*30*
" "	*Bills receivable,*	*1*	*200*	*00*					
			974	*70*				*974*	*70*

88 Only 9 dates and 30 items will appear in the two pages of the Day Book. Six of these items will be posted in the Cash account. Other entries of the Cash account include interest collections, "sales to date," and cash sales. B's and C's Ledger accounts and the Balance Sheet will be written on the fourth page. The debit side of the Balance Sheet should be: Mdse. on hand, $820; Cash on hand, $101.67; Bills Receivable, $150; Bernard Frost (B), $4.28; Amos Lawrence (A), $11.70. The credit side: Bills Payable, $260.00; Charles Smith (C), $31.95; Net capital, $795.70.

89 After the first four items under the first date, Apr. 1, the following entries of the Day Book should be made in order: First page: C, 2 items; A, 3 items; B, 3 items; A, 1 item; C, 1 item. Second page: B, 3 items; C, 3 items; A, 2 items; C, 3 items; A, 1 item; A, 3 items. In the Cash account, there will be 9 entries on the debit side and 13 entries on the credit side. C's Ledger account will also be on the third page of the sheet.

Balance Sheet Dr.: Mdse. on hand, \$3000.00; Cash on hand, \$2473.38; Amos Lawrence (A), \$280.60; Bills Receivable, \$250. Cr.: Charles Smith (C), \$761.25; Net capital, \$5242.73.

90 This account is supposed to extend from March 1 to March 19. The cash balance March 9 is \$113.04, and May 19, \$199.60. The Balance Sheet is as follows: Dr.: Cash on hand, \$199.60; Mdse. on hand, \$400; Byron Waters, \$40.00. Cr.: Dennis Rand, \$4.78; Net capital, \$634.82.

92 *Answers:* **3** 26 56. **4** $f \pm [d \times (n-1)]$. **5** 50 39 10. **6** $d = \frac{l-f}{n-1}$, or, $\frac{f-l}{n-1}$; $n = \frac{l-f}{d}$, or, $\frac{f-l}{d}$. **7** 5. **8** 6. **10** 108. **11** $S = \frac{f+l}{2} \times n$. **12** 420. **13** \$1.45 \$19.50.

It would be well to place upon the board several series of numbers like the following:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	3	5	7	9	11	13	15
4	8	12	16	20	24	28	32
30	27	24	21	18	15	12	9

Lead the pupils to discover the fact that in each of the above series of numbers there is a constant difference between the consecutive terms, and to make a definition like the following: Arithmetical progression is a series of numbers which increase or decrease by a constant difference.

Any term of an arithmetical series may be found by multiplying the common difference by the number of terms which preceded it.

The other rules may be readily found from the given examples. To teach the rule for finding the sum of an arithmetical series, place upon the board any series, as :

3 6 9 12 15 18

and, directly below it, the same series reversed. The two series will appear as follows :

3	6	9	12	15	18
21	18	15	12	9	6
24	24	24	24	24	24

= twice the sum of the series. Therefore the rule : The sum of an arithmetical series is equal to the product of one half the sum of the first and last terms multiplied by the number of terms.

93 *Answers :* **6** 2430. **7** 60¾. **8** 5. **10** 98304. 131070. **11** $25.60 $51.15. **12** $59049 $88573. **13** $179.08. **14** 9 yr. 2 da. **15** $10737418.21.

First place upon the board two or more series of numbers in geometrical progression, as follows :

2	6	18	54
40	20	10	5

The pupils will see that each of the above series of numbers increases or diminishes by a constant ratio. Geometrical progression, therefore, is a series of numbers which increase or diminish by a constant ratio.

To show how any term or ratio of a geometrical series may be found, write the factors of each term of the series upon the board ; thus :

2	6	18	54
2	2×3	$2 \times 3 \times 3$	$2 \times 3 \times 3 \times 3$

By questioning the pupils upon the above numbers, the following rules may be developed :

The last term of a geometrical series is equal to the first term multiplied by the ratio raised to a power whose degree is one less than the number of terms. The first term is equal to the last

term divided by the ratio raised to a power whose degree is one less than the number of terms.

The ratio is equal to the root whose index is one less than the number of terms, of the quotient of the last term divided by the first term.

The sum of the series may be found as indicated in **9**. Other formulas may be expressed as follows:

$$l = f \times r^{n-1};\ f = \frac{l}{r^{n-1}};\ r = \sqrt[n-1]{l \div f}.$$

94 *Answers:* **1** $6\frac{3}{4}¢$. **2** $41\frac{7}{15}¢$. **3** 2 lb. @ 9¢, 1 lb. @ 6¢ 4 lb. @ 6¢, 4 lb. @ 9¢. **4** 1 lb. @ 50¢, 1 lb. at 65¢, $2\frac{1}{4}$ lb. @ $1. **5** 15 lb. **6** 20 lb., 20 lb., 60 lb. **7** $12\frac{1}{2}$ gal. **8** 1 lb. $2\frac{2}{3}$ oz. **9** 1 lb. 11 oz. 11 pwt. 5.28 gr. **10** 3.98 g. **11** 6 oz. **12** 3 oz. 5 pwt. **13** $2.45. **14** $3\frac{1}{7}\%$. **15** 23.52 oz. 49 lb.

Many of the questions and answers given on the remaining pages were given by business men, mechanics, or specialists.

95 *Answers:* **1** $39110. **2** $3600 loss. **4** $361.07. **5** $545.43. **6** $216.01. **7** 85 lb. 89 lb. **8** Jan. 15. **9** About $4\frac{1}{2}\%$. **10** 5% stock $\frac{1}{4}\%$ better.

96 *Answers:* **1** $116.68. **2** $172.72. **3** $392.20. **4** $112\frac{1}{4}$ $75 5.347%. **5** $8.22. **6** $26.44. **7** 29531 bricks. **8** 5276.97984 ft. **9** $21.56. **10** $.128+.

97 *Answers:* **1** $5.472. **2** Rivets, $\frac{5}{8}$ lb; burrs, $\frac{1}{8}$ lb. **3** 650.4 yd. **4** $172.40. **5** $850. **6** $5\frac{1}{2}\frac{5}{2}\%$ gain $3\frac{9}{2}\frac{9}{2}\%$ loss. **7** $11.12 18 rolls 14 yd.

98 *Answers:* **1** (*a*) 8200 ft.; (*b*) 7520 ft.; (*c*) 4700 ft.; (*d*) 151 bundles; (*e*) 108 bundles; (*f*) 660 ft.; (*g*) $591.39. **2** $87936.51. **3** A, $9000 B, $10125 C, $7875. **4** $811.69. **5** 125 bu. 140 bu. **6** (*a*) 374 yd.; (*b*) 1 A. 144 rd.; (*c*) 80 sq. rd.; (*d*) 108 sq. rd.; (*e*) 116 sq. rd.

99 *Answers:* **1** $2\frac{9}{3}\frac{9}{8}$ cd. $\frac{3}{8}\frac{8}{5}$ cd. **2** 105° 15′. **11** 720 pass-books. **12** Mercury, $5\frac{3}{7}$ as large as the moon. **14** 1350 sq. in. $1633\frac{1}{2}$ sq. in. **15** 8.944+ rd. 8 rd. 15.576+ ft.

100 *Answers:* **1** 66 rd. 5 ft. 3 A. 5 sq. rd. 12 sq. yd. 7 sq. ft. 72 sq. in. **2** \$408.41. **3** \$44.12. **4** 1051 bricks. **5** 75.2 65.2 32.2 31.2 27.2 16.22 15.01 11.94. **6** 41 strips 13 rolls \$2.44 37 yd. **7** 646.5+ bbl.

101 *Answers:* **1** \$10150. **2** 253 sq. ft. 18 sq. in. **3** \$120. **4** 4096 bullets. **5** A, \$512.82 B, \$687.18. **6** 20 yd. **7** July 16. **8** *ps pi pgy ptt ara.* **9** $11\frac{1}{9}\%$ gain $11\frac{1}{9}\%$ loss 8% loss 15.1% gain $42\frac{6}{7}\%$ gain. **10** 41.4+ bbl. **11** 56.57+ gal.

102 *Answers:* **3** 57+% 28+% 68+%. **4** N.Y., 1123385932. **5** S. Dakota, 1048+%. **6** 1880: 16983034673; 1890: 25265581794.

103 *Answers:* **1** 18 h. 37 min. 10 sec. **2** $5\frac{3629}{3630}$ A. **3** $4\frac{533}{880}$ Eng. mi. **4** 4071 mi. **5** Wheat, $530\frac{2}{5}$ bu.; apples, 442 bu.; beets, 442 bu.; carrots, 442 bu.; potatoes, 442 bu.; corn, 221 bu.; salt, $230\frac{2}{5}$ bu. **6** 80 perches. **7** $567\frac{3}{16}$ 6171 T. **8** Wheat, $33\frac{1}{3}$ bu.; rye, $35\frac{5}{7}$ bu.; oats, $57\frac{1}{2}$ bu.; barley, $41\frac{2}{3}$ bu.; salt, 40 bu.; potatoes, $33\frac{1}{3}$ bu.; coal, 25 bu. **9** $107\frac{1}{7}$ bu. **10** $6\frac{1}{8}$ lb. **11** \$26.67. **12** \$10 gain. **13** 600 lb. **14** \$13.14. **15** Radius, 120 in.

In **5**, the approximate answers are given, the bushel being reckoned as containing $1\frac{1}{4}$ cu. ft.; heaped bushel, $1\frac{1}{2}$ cu. ft., and 2 bu. ears as equal in bulk to 1 bu. shelled corn.

104 *Answers:* **1** Pop. Sweden, 4784675; pop. U.S., 62622250; children, Belgium, 827928; rate p.c., Bavaria, 21.2; rate p.c. Neth., 14.4. **2** £3906 5*s*. **3** \$500.89. **4** About 13 m. **5** 18+%. **6** 44+%. **7** \$1600. **8** 4 h. 6.54+ min. P.M. **9** 50% 5%.

105 *Answers:* **1** \$504.07. **2** 2d is greater discount by 2.85%; \$22.80 saved. **3** \$9.75 gain. **4** $\$436\frac{4}{11}$ $\$163\frac{7}{11}$. **5** 59.48+ gal. **6** 1656 mi. **7** $10\frac{5}{9}$ h. **8** 2096.11 \$1855.59 \$2511.22. **9** $83\frac{1}{8}$ ft. **10** $156\frac{1}{4}\%$ premium.

106 *Answers:* **1** 33 min. 27.5+ sec. 15 min. 40.1+ sec. 33 min. 16.8+ sec. **2** 78.5+ bu. 133.5+ bu. 307.9+ bu. **3** 1302.3+ bu. **4** 890.1+ gal. **5** 1692.02 gal. 35.7+ in. **6** 91.19+ bbl. **9** .019+^{mm}. **10** 1.655012$^{cu\ cm}$ 1.04+%.

107 *Answers:* **1** .49 3.94%. **2** 3045 meters. **3** 59.79 times. **4** 47.31 in. 9.775 in. **5** 6.316 sec. **6** 144.72 ft. 305.52 ft. 466.32 ft. **7** 3618 ft. **8** 188.94 ft. per sec.

108 *Answers:* **1** 4 ft. **2** $11\frac{1}{4}$ in. from end where the Kilogram weight is. **3** $6\frac{3}{4}$ in. from middle on same side as 2 and 3. **4** 129.16^{g}. **5** 8 lb. on balance near the weight, and 4 lb. on the other. **6** 11 lb. on balance near the 12 lb. weight; 10 lb. on other balance.

Many solutions are possible for **7**; *e.g.* 60 lb. in middle, and 40 lb. $2\frac{1}{2}$ ft. from man; or, 40 lb. in middle, and 60 lb. $3\frac{1}{3}$ ft. from man.

109 *Answers:* **1** 49+ lb., *i.e.* anything in excess of 49 lb. **2** Anything in excess of $\frac{1}{3}$ of person's weight. **3** 25+ lb. **4** 20+ lb. **5** 73.5+ lb. **6** 18849600 lb. 2282568.75 lb. **7** $\frac{5}{10}$ $\frac{5}{11}$ $\frac{1}{4}$.

110 *Answers:* **1** 60 lb. **2** 58.92 lb. **3** .84 lb. **4** 12 boys 12 boys. **5** Boxwood, .537+; maple, .463+. **6** 7.29. **7** 1.125. **8** 2.56 2.08^{mm}. **9** 1.13+. **10** .0000^{mg}. **11** 26.32 ft.

111 *Answers:* **2** .565+. **3** .376. **4** .307. **5** 264 297 330 352 396 440 495 528. **6** About 43°. **7** About 20 millions of millions of miles. **8** 12.7+.

The computed result of **1** is 18.99+^{cm}. The difference may be disregarded, as it is not greater than probable error in reading scale used.

In **8**, sound is reckoned as traveling 1120 ft. per second.

112 *Answers:* **1** A little more than 116°. **2** 20.78 in. **3** 3.144 ohms. **4** 100.61 ohms 40.24 ohms. **5** $\frac{1}{2}$ its weight at surface $\frac{1}{4}$ its weight at surface 0. **6** 2800 mi. from centre. **7** 25 lb. $11\frac{1}{9}$ lb. 4 lb. $11\frac{1}{9}$ lb. $6\frac{1}{4}$ lb. **8** 24000 mi.

113 *Answers:* **1** 20.8+ sec. **2** $1\frac{3}{4}\frac{3}{4}$ times as loud. **3** 4.99+ qt. **4** 12.58+ pk. **5** 34.37+ ft. **6** 2500 lb. 1.55+ ft. **7** 1 ft. 1.1+ in. **8** About 117 bbl. **8** 2160° F. **10** 2880° F. **11** 787.92 ft. **12** 3.61+ sec. 7.82+ sec. **13** 9.775 in. 156.4 in. **14** 2.9+ times a second.

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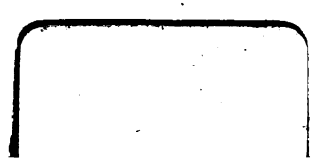

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